Released February 2018

Years 5/6

Small Steps Guidance and Examples

Block 3

White Rose Maths

Year 5/6 – Yearly Overview

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Number – Place Value**			**Number – Addition and Subtraction**		**Number – Multiplication and Division**			**Statistics**		**Measurement: Perimeter, Area and Volume**	
Spring	**Number – Fractions**					**Number- Decimals and Percentages**				**Year 5: Multiplication and Division Year 6: Algebra and Ratio**		
Summer	**Measurement: Converting Units**		**Geometry: Position and Direction**	**Geometry: Properties of Shape**			**Investigations**					**Consolidation**

Overview

Small Steps

Year 5	Year 6
Prime Numbers	
Square Numbers	
Cube Numbers	
For the rest of this block, Year 5 should recap their learning from Autumn and Spring term. This time can be used to fill any gaps in learning as well as deepen understanding through more reasoning and problem solving activities.	Find a rule – one step
	Find a rule – two step
	Use an algebraic rule
	Substitution
	Formulae
	Word Problems
	Solve simple one step equations
	Solve two step equations
	Find pairs of values
	Enumerate possibilities

Overview

Small Steps

Year 5	Year 6
For the rest of this block, Year 5 should recap their learning from Autumn and Spring term. This time can be used to fill any gaps in learning as well as deepen understanding through more reasoning and problem solving activities.	Using ratio language
	Ratio and fractions
	Introducing the ratio symbol
	Calculating ratio
	Using scale factors
	Calculating scale factors
	Ratio and proportion problems

Prime Numbers

Notes and Guidance

Using their knowledge of factors, children see that some numbers only have 2 factors and these are special numbers called Prime Numbers. They also learn that non-primes are called composite numbers. Children can recall primes up to 19 and are able to establish whether a number is prime up to 100.

Using primes, they break a number down into its prime factors.

Mathematical Talk

How many factors does each number have?

How many other numbers can you find that have this number of factors?

What is a prime number?

What is a composite number?

How many factors does a prime number have?

Varied Fluency

1. Use counters to find the factors of the following numbers.

   5, 13, 17, 23

   What do you notice about the arrays?

2. A prime number has 2 factors, one and itself. A composite number can be divided by numbers other than 1 and itself. Sort the numbers into the table.

   5 15 9 12 3 27 24 30

	Prime	Composite
2 factors (1 & itself)		
More than 2 factors		

Put two of your own numbers into the table. Why are two of the boxes empty?
Where would 1 go in the table? Would it fit in at all?

Prime Numbers

Reasoning and Problem Solving

Find all the prime number between 10 and 100, Sort them in the table below.

End in a 1	End in a 3	End in a 7	End in a 9

What is the same about the groups?

Why do no two-digit prime numbers end in an even number?

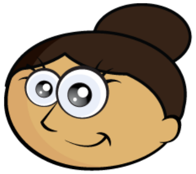
Why do no two-digit prime numbers end in a 5?

End in a 1	End in a 3
11, 31, 41, 61, 71,	13, 23, 43, 53, 73, 83
End in a 7	**End in a 9**
17, 37, 47, 67, 97	19, 29, 59, 79, 89

No 2-digit primes end in an even number because 2-digit even numbers are divisible by 2.

No 2- digit prime numbers end in a 5 because they are divisible by 5 as well as 1 and itself.

Katie says all prime numbers have to be odd.



Her friend Abdul That means 9, 27 and 45 are prime numbers.

Explain Abdul and Katie's mistakes and correct them.

2 is a prime number so Katie is wrong. Abdul thinks all odd numbers are prime but he is wrong as the numbers he has chosen have more than 2 factors.

9= 1, 3 & 9 as factors

27 = 1, 3, 9 & 27

45 = 1, 3, 5, 9, 15 & 45

Always, sometimes, never

The sum of two prime numbers is even.

Sometimes: The sum of any 2 odd prime numbers is even. However if you add 2 and another prime number your answer is odd.

Square Numbers

Notes and Guidance

Children will need to be able to find factors of whole numbers. Square numbers have an odd number of factors and are the result of multiplying a number by itself.

They learn the notation for squared is ^{2}.

Mathematical Talk

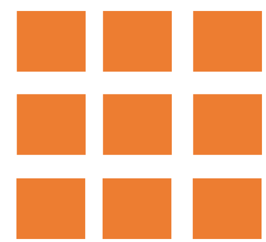
Why are square numbers called ‘square numbers?

Is there a pattern between the numbers?

True or False: The square of an even number is even and the square of an odd number is odd

Varied Fluency

1. What does this array show you?.
   Why is it square?



2. How many ways are there of arranging 36 counters?
   Explain what you notice about the different arrays.
   How many different squares can you make using counters?
   What do you notice?
   Are there any patterns?

3. Find the first 12 square numbers.
   Prove that they are square numbers.

Square Numbers

Reasoning and Problem Solving

Chris says  Factors come in pairs so all whole numbers must have an even number of factors. Do you agree? Explain your reasoning.	Children will find that some numbers don't have an even number of factors e.g. 25. Square numbers have an odd number of factors.
How many square numbers can you make by adding prime numbers together? Here's one to get you started: $2 + 2 = 4$.	Solutions include: $2 + 2 = 4$ $2 + 7 = 9$ $11 + 5 = 16$ $23 + 2 = 25$ $29 + 7 = 36$

Julian thinks that 4^2 is equal to 16. Do you agree? Convince me. He also thinks that 6^2 is equal to 12. Do you agree? Explain what you have noticed.	Children may use concrete materials or draw pictures of to prove it. Children should spot that 6 has been multiplied by 2. They may create the array to prove that $6^2 = 36$ and $6 \times 2 = 12$
Always, Sometimes, Never: A square number has an even number of factors.	Never. Square numbers have an odd number of factors.

Cube Numbers

Notes and Guidance

Children learn that a cubed number is the product of three numbers which are the same.

If you multiply a number by itself, then itself again the result is a cubed number.

They learn the notation for cubed is ^{3}

Mathematical Talk

How are squared and cubed numbers the same?

How are they different?

True or False: Cubes of even numbers are even and cubes of odd numbers are odd

Varied Fluency

1. Use multilink cubes and investigate how many are needed to make different sized cubes.

How many multilink cubes are required to make the first cubed number? The second? Third?

Can you predict what the tenth cubed number is going to be?

2. Complete the following table.

3^3	$3 \times 3 \times 3$	27
5^3	$5 \times 5 \times 5$	
	$6 \times 6 \times 6$	
4^3		
		8

3. Calculate:

3^3=

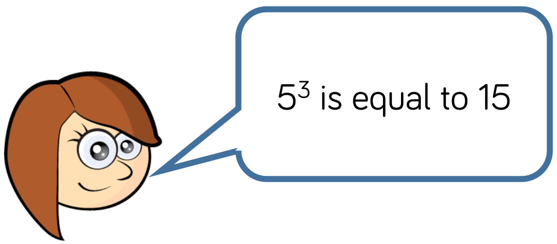
5^3=

4 cubed=

6 cubed=

Cube Numbers

Reasoning and Problem Solving

Lisa says.  Is she correct?	No- She has multiplied 5 times three rather than 5 times 5 times 5
Here are 3 number cards A B C Each number card is a cubed number. Use the following information to find each number $A \times A = B$ $B + B - 3 = C$ Digit total of C = A	A = 8 B = 64 C = 125

Jenny is thinking of a two-digit number that is both a square and a cubed number. What number is she thinking of?	64
Caroline's daughter has an age that is a cubed number. Next year her age will be a squared number. How old is she now?	8
The sum of a cubed number and a square number is 150. What are the two numbers?	125 & 25

Find a Rule – One Step

Notes and Guidance

Children begin by exploring simple one step function machines. Explain that a one-step function is where they perform just one operation on a particular input value.
Children understand that for each number they put into a function there is an output.
Children should be able to write these one step functions as simple algebraic expressions. They should understand that we write simple functions such as $a \times 4$ as $4a$.

Mathematical Talk

What do you think one-step function means?
What examples of functions do you know? What do you think input and output means?
What is the output if ….?
What is the input if ….?
Work out the function if you know the following functions?
How many sets of input and output do you need to be able to work out the function? Explain your answers.
What is the algebraic rule for that function machine …?

Varied Fluency

1. Here is a function machine.

Input $\longrightarrow$ $\boxed{\times 4}$ $\longrightarrow$ Output

- What is the output if the input is 2?
- What is the output if he input is 7.2?
- What number went in if the output was 22?
- What is the output if the input is a? What about if you put x in?

2. Complete the table for the given function machine.

Input $\longrightarrow$ $\boxed{+ 5}$ $\longrightarrow$ Output

Input	5	5.8	10	-3	-8				a	y
Output						9	169	0		

3. Write your function as an algebraic rule?
Work out the functions

$10 \longrightarrow \boxed{} \longrightarrow 5$
$24 \longrightarrow \boxed{} \longrightarrow 12$
$7 \longrightarrow \boxed{} \longrightarrow 3.5$

Find a Rule – One Step

Reasoning and Problem Solving

Meg has a one-step function machine. She puts in the number 6 and the number 18 comes out. 6 → [ ] → 18 What could the function be? How many different answers can you find?	The function could be +12, ×6, subtract from 24, divide by 1/3
Giles puts in some numbers into a function machine. 2 → [ ] → 8 3 → [ ] → 7 6 → [ ] → 4 What is he output from the function when the input is 16?	The function is subtract from 10 so the output is -6

Lucy is using the following function machine. Input → ÷ 2 → Output Lucy put a number into the machine. She puts the output back into the machine and gets out another number. The final answer is 2.5 What number did Lucy put in?	10
Lucy has another function machine. • She puts a number 8 and gets an output. • She puts the output back into the machine. • The final output is -6 What could the function be?	Subtract 7 (- 7)

Find a Rule – Two Step

Notes and Guidance

Children build on their knowledge of one-step functions to consider now two-step function machines. Discuss with children whether a function such as +5 and + 6 is a two-step function machine or whether it can be written as a one-step function. They look at strategies to find the functions, given a series of inputs and outputs. They do this by trial and error or by considering the pattern of differences. Children record their input and output values in the form of table.

Mathematical Talk

How can you write + 5 followed by – 2 as a one-step function?
Do the functions have to be different?
If I switched around the functions, do you get the same answers?
What is the output if ….? What is the input if ….? How did you work it out?
What is the function machine if a is the input and $3a - 2$ comes out.
What method did you use to find a two-step function?

Varied Fluency

1. Here is a function machine.

Input $\longrightarrow$ $\boxed{\times 2}$ $\longrightarrow$ $\boxed{+5}$ $\longrightarrow$ Output

- What is the output if the input is 5?
- What number went in if the output was 19
- What is the output if the input is a? What about if you put x in?

2. Complete the table for the given function machine.

Input $\longrightarrow$ $\boxed{\times 3}$ $\longrightarrow$ $\boxed{-4}$ $\longrightarrow$ Output

Input	1	2	3	4	5
Output					

- What patterns do you notice in the outputs?
- What is the input if 20 comes out? How did you work it out?
- What is the algebraic function for this function?

3. What does the function machine look like for each of these algebraic expressions?

$\boldsymbol{a \times 3 + 2}$ $\boldsymbol{5a - 4}$ $\boldsymbol{(a + 3) \times 2}$

Find a Rule – Two Step

Reasoning and Problem Solving

David has a two function machines.

Input → $+5$ → $\times 2$ → Output

Input → $\times 2$ → $+5$ → Output

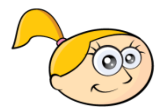
He says,

The function machines will give the same answer.

Is David correct?
Is there an input that will give the same output for both machines?

No they do not give the same answer. Children can just put any number in to show this.

Lola has the following function machine.



Input → $+2$ → -8 → Output

Explain how this can be written as a single function machine.

Write is as −6

Use an Algebraic Rule

Notes and Guidance

Children have now met one-step and two-step function machines and have formed expressions from these machines. Children are now presented with abstract algebraic expressions and have to work out the one-step or two-step rules.
They again work out input and output values given the rule, not realising at this stage that they are doing formal substitution and solving equations. Children need to be able to recognise standard simplified versions of the rules.

Mathematical Talk

What expressions can be formed from this function machine?

What would the function machine look like for this rule/expression?

How can you write $a \times 3 + 5$ differently?

Are $2a + 6$ and $6 + 2a$ the same? Explain

Varied Fluency

1. What algebraic rules come from the following function machines?

   Input → $\boxed{\times 5}$ → Output

   Input → $\boxed{\times 2}$ → $\boxed{-5}$ → Output

   What is the output when 7 is input into each function machine?

2. What does the function machine look like for each of these algebraic expressions?

$a \times 4$	$a + 10$	$a - 10$
$a \div 2$	$3a$	$a \div 5$
$a \times 5 + 3$	$3a - 1$	$(a + 3) \times 2$

   What is the output when 7 is substituted into each of these functions?

3. Here is an expression that has come from a function machine: Output = $4a + 3$

   - What is the output if the input, a, is 7?
   - What is the output if he input is 2.5?
   - What is the input if the output is 63?

Use an Algebraic Rule

Reasoning and Problem Solving

These two function machines give the same answer.

Input → × 2 → +8 → Output

Input → [ ] → × 2 → Output

What is missing part of the function?
Can you explain why using base 10?
Using algebraic expressions?

+ 4
Because a x 2 + 8 is the same as (a + 4) x 2

This function machine gives the same output for every input.
For example if the input is 5 then the output is 5 and so on.

Input → × 2 → [ ] → Output

What is the missing part of the function?
What other pairs of functions can you think that will do the same?

÷2
Functions that are the inverse of each other.

Complete the table of values for the following function.

$$5 + 2a$$

Input	5	7		
Output			75	30

15, 19, 35, 12.5

Substitution

Notes and Guidance

Children substitute into simple expressions and equations to find a particular value.

They have already experienced substitution in a less formal way and allowing children to see the link between this and formal substitution will help it feel less abstract.

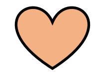
Mathematical Talk

Which letter represents the star? Which letter represents the heart?
Would it still be correct if it changed to $a + b + c$?

What do you notice about your final answer in question 2 and your first answer in question 3?

What does it mean when a number is next to a letter?

Varied Fluency

1. If  = 7  = 5 what is the value of:

What is the same and what is different about this question?
If $a = 7$ and $b = 5$ what is the value of:

$$a + b + b$$

2. Substitute into the following expressions when,

$$\boxed{w = 3 \qquad x = 5 \qquad y = 2.5}$$

- $w + 10$
- $w + x$
- $y - w$
- $w + x + y$
- $w - x - y$
- $y + y + y$

3. Substitute into the following expressions when,

$$\boxed{w = 10 \qquad x = \frac{1}{4} \qquad y = 2.5}$$

- $3y$
- wx
- $12 + 8.8w$
- $x \times (w + 2y)$

Substitution

Reasoning and Problem Solving

Here are two equations.

$$p = 2a + 5$$

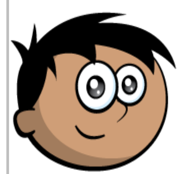
$$c = 10 - p$$

Find the value of c when $a = 10$

$c = -15$

$$x = 2c + 6$$

Joe says,

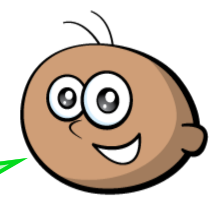


$x = 12$ because c must be equal to 3 because it's the 3rd letter in the alphabet

Is Joe correct?

Derek says,

When $c = 5$ the answer is 31



Is Derek correct?

No Joe is incorrect. c could have any value.

No Derek is incorrect, he has just put the 2 and 5 together to make 25 instead of multiplying them.

Formulae

Notes and Guidance

Children substitute into familiar formulae such as the formula for area and volume.

They also use simple formulae to work out values of everyday activities such as the cost of a taxi or the amount of medicine to take given a person's age.

Mathematical Talk

What tells you something is a formula?

In the formula $C = £1.50 + 0.3m$ what do you think the 'C' stands for?
What do you think the m stands for?

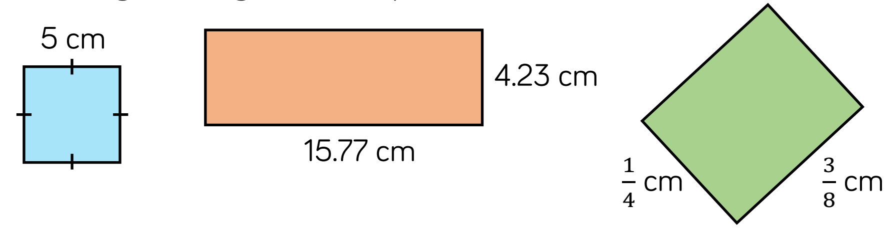
Varied Fluency

1. Tick the formula.

$P = 2(l + w)$ | $3d + 5$ | $20 = 3x - 2$

Explain why the other two are not formulae.

2. Substitute into $P = 2(l + w)$ to find the perimeter of the following rectangles and squares.



Use the formula for area of a rectangle to also find the area.

3. This is the formula to work out the cost of a taxi.

$$C = 1.50 + 0.3m$$

$m =$ number of miles travelled.
Work out the cost of the taxi when it travels 12 miles.

Formulae

Reasoning and Problem Solving

Joe and Nadia are using the following formula to work out what they should charge for four hours' cleaning. Cost in pounds = 20 + 10 × number of hours Joe writes down £60 Nadia writes down £120 Who do you agree with? Why?	Joe is correct as multiplication should be performed first. Nadia has not used the order of operations.
The rule for making scones is use 4 times as much flour (f) as butter (b). Which is the correct formula to represent this? A $f = \frac{b}{4}$ B $f = 4b$ C $f = b + 4$ D $4f = b$ Explain why the others are incorrect.	B is correct.

Word Problems

Notes and Guidance

Children begin to start thinking about solving equations through worded problems.

This helps children see a reason for solving an equation and gives them something to relate more abstract equations to.

Ensure to use concrete materials when first introducing this concept.

Mathematical Talk

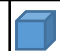
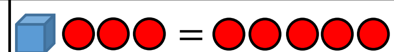
What does the cube represent?
What do the counters represent?

Can you think of your own 'think of a number' problems?

Why are the questions in Q3 more difficult to represent using concrete materials?

Varied Fluency

1. Here is a word problem represented with concrete resources and algebra.

Words	Concrete	Algebra
I think of a number		x
Add 3		$x+3$
My answer is 5		$x+3=5$

Can you complete this table?

Words	Concrete	Algebra
I think of a number		
Add 1		
My answer is 8		

2. Use concrete materials to represent these equations.

$w+4=7$ $10=2+t$ $3+x=9$

3. Write the algebra to match the sentences.

- I think of a number, subtract 17, my answer is 20
- I think of a number, multiply it by 5, my answer is 45

Word Problems

Reasoning and Problem Solving

Jane thinks of a number, she adds 7 and divides her answer by 2 Mike thinks of a number, he multiples by 3 and subtracts 4 Jane and Mike think of the same number. Jane's answer is 9 What is Mike's answer? Jane and Mike think of the same number again and they both get the same answer. Use trial and error to find the number they were thinking of	They both think of 11, therefore Mike's answer is 29 They think of 3 and the answer they both get is 5

Kira spends 92p on yoyos and sweets She buys y yoyos costing 11p and s sweets costing 4p. Can you write an equation to represent what Kira has bought? How many yoyos and sweets could Kira have bought? Can you write a word problem to describe this equation? $74 = 15t + 2m$	$92 = 11y + 4s$ She could have bought 1 sweet and 8 yoyos or 4 yoyos and 12 sweets

One Step Equations

Notes and Guidance

Children solve simple one step equations involving the four operations.

Children should explore and build on the use of concrete materials such as cubes, counters and cups.

It is recommended that children learn to solve equations using a balancing method and the use of inverse operations.

Mathematical Talk

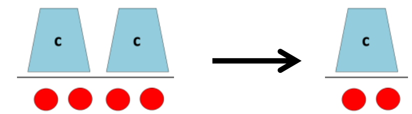
Can you make some of your own equations using cups and counters for a friend to solve?

Why do you think the equation is set up on a balance? What does the balance represent? How does this help you solve the equation?

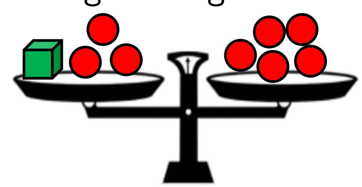
What is the same and what is different about each bar model?

Varied Fluency

1. What's in the cup?
Write down and solve the equation represented by the diagram.



2. Solve the equation represented on the scales.
Can you draw a diagram to go with the next step?

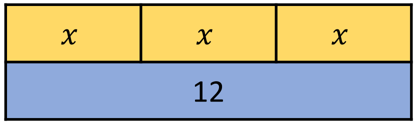


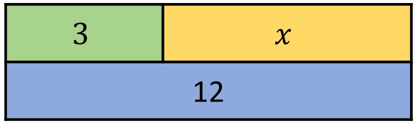
3. Match each equation to the correct bar model then solve.

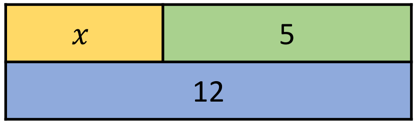
$x + 5 = 12$

$3x = 12$

$12 = 3 + x$

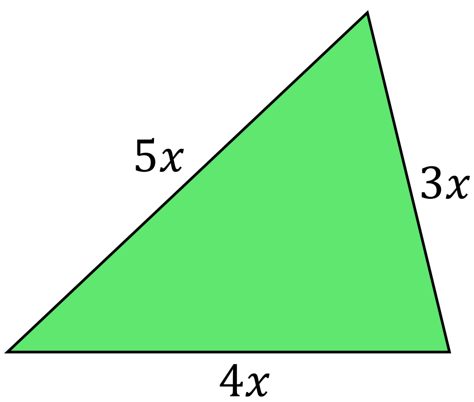


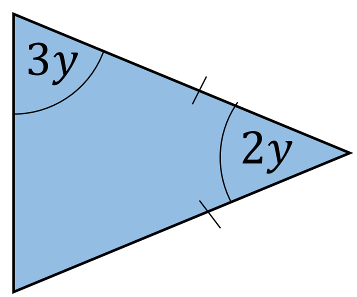




One Step Equations

Reasoning and Problem Solving

The perimeter of the triangle is 216 cm.  Work out each length of the triangle by forming and solving an equation then substituting your answer.	$12x = 216$ $x = 18$ $5 \times 18 = 90$ $3 \times 18 = 54$ $4 \times 18 = 72$

• Hannah is 8 years old • Jack is 13 years old • Grandma is $x + 12$ years old. • The sum of their ages is 100 Form and solve an equation to work out how old Grandma is.	$8 + 13 + x + 12 =$ 100 $33 + x = 100$ $x = 67$ Grandma is 79 years old.
What is the size of the smallest angle in this isosceles triangle?  How can you check your answer?	$8y = 180$ $y = 22.5$ Smallest angle = 45° Check by working them all out and see if they add to 180°

Two Step Equations

Notes and Guidance

Children progress from solving equations that require one step to equations that require two steps.

Children should think of each equation as balance and solve it through doing the same thing to each side of the equation.

Mathematical Talk

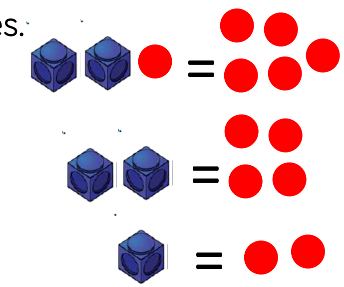
Why do you have to do the same to each side of the equation?

Why subtract 1? What does this do to the left hand side of the equation?

Does the order the equation is written in matter?

Varied Fluency

1. Here is each step of an equation represented with concrete resources.



$$2x + 1 = 5$$
$$-1 \qquad -1$$
$$2x = 4$$
$$\div 2 \qquad \div 2$$
$$x = 2$$

Use this method to solve:

$4y + 2 = 6$ $\qquad 9 = 2x + 5$ $\qquad 1 + 5a = 16$

2. Solve the following equations.

$3y + 5 = 26$ $\qquad 10 = 17 + 2x$

$0.5w - 1 = 0$ $\qquad 2q - \frac{1}{6} = 6 - \frac{1}{6}$

$114 = \frac{y}{5} + 99$ $\qquad 10 - 2x = 4$

Two Step Equations

Reasoning and Problem Solving

The length of a rectangle is $2x + 3$
The width of the same rectangle is $x - 2$
The perimeter is 17 cm

Find the area of the rectangle.

$6x + 2 = 17$
$6x = 15$
$x = 2.5$
Length = 8 cm
Width = 0.5 cm
Area = 4 cm^2

Katy has some algebra expression cards.

$y + 4$ | $2y$ | $3y - 1$

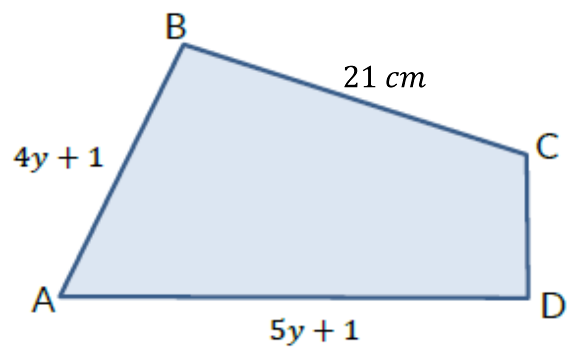
The mean of the cards is 19
Work out the value of each card.

$6y + 3 = 57$
$6y = 54$
$y = 9$

Card values:
13
18
26

The diagram shows a quadrilateral ABCD.

The perimeter of the quadrilateral is 80 cm.



AB is the same length as BC.

Find the length of CD.

$4y + 1 = 21$
$4y = 20$
$y = 5$
AB = 21 cm
BC = 21 cm
AD = 26 cm
CD = 12 cm

Find Pairs of Values

Notes and Guidance

Children will use their understanding of how to solve an equation and apply this knowledge to finding the possible values of a pair.

Mathematical Talk

What is the question asking you to do?

How many possible answers are there? Convince me you have them all.

What do you notice about the values of a and b?

Varied Fluency

1. a and b are variables:

$$a + b = 6$$

Find 5 different possible values for a and b.

a	b

2. X and Y are whole numbers.
   - X is a one digit odd number.
   - Y is a two digit even number.
   - $X + Y = 25$

   Find all the possible pairs of numbers that satisfy the equation.

3. $$a \times b = 48$$

   What is the value of a and b?
   How many different ways can you find?

Find Pairs of Values

Reasoning and Problem Solving

a, b and c are integers between -5 and 5

$$a - b = -3$$
$$b + c = 3$$

Find the values of a, b and c
How many different possibilities can you find?

Use the possible values to complete the equation:

$$a + c = \square$$

Possible answers:

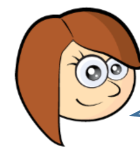
a = -5 b = -2
c = 5
a = -4 b = -1
c = 4
a = -3 b = 0
c = 3

a + c = 0

x and y are both positive whole numbers.

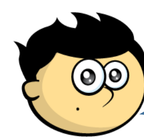
$$\frac{x}{y} = 4$$

Jade says,



x will always be a multiple of 4

Simon says,



y will always be a factor of 4

Who is correct?
Prove it!

Possible answer:
Jade is correct as x will always have to divide into 4 equal parts. E.g, 32 ÷ 8 = 4, 16 ÷ 4 = 4
Simon is incorrect. 40 ÷ 10 = 4 and 10 is not a factor of 4

Enumerate Possibilities

Notes and Guidance

Children see they can enumerate possibilities (list of possibilities)

They need to use number properties efficiently to satisfy a specific criteria that is set.

Mathematical Talk

What does $2a$ mean? (2 multiplied by an unknown number)
What is the greatest/smallest number 'a' can be?

What strategy did you use to find the value of 'b'?

Can you draw a bar model to represent the following equations:
$3f + g = 20$
$7a + 3b = 40$
What could the letters represent?

Varied Fluency

1. In this equation, a and b are both whole numbers which are less than 12.

$$2a = b$$

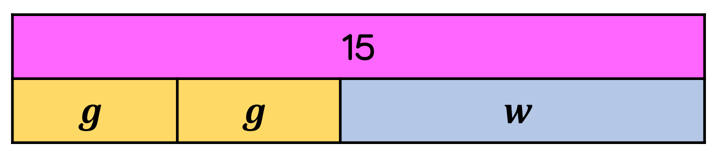
Write the calculations that would show all the possible values for a and b.

2. Use the equation to fill in the missing values in the table below.

$$7x + 4 = y$$

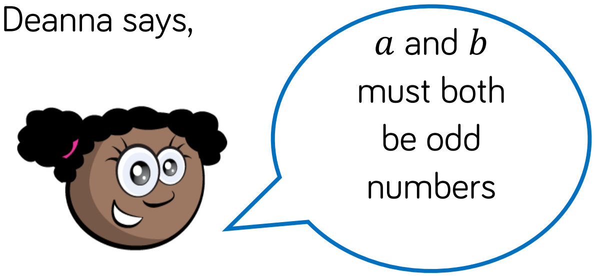
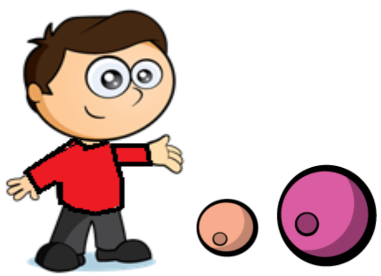
Value of x	Value of y

3. $2g + w = 15$
Write down all the possible values for g and w, show each of them in a bar model.



Enumerate Possibilities

Reasoning and Problem Solving

$ab + b = 18$ Deanna says,  Is Deanna correct? Prove it.	Possible answer: Deanna is incorrect. Children may give examples to prove Deanna is correct. For example, $5 \times 3 + 3 = 18$. But there are also examples to prove she is incorrect. E.g. $2 \times 6 + 6 = 18$ where a and b are both even.
Large beads cost 5p and small beads cost 4p Mr Smith has 79p to spend on beads.  How many different combinations of small and large beads can Mr Smith buy? Can you write expressions that show all the solutions?	Possible answers: $11l + 6s$ $7l + 11s$ $15l + s$ $3l + 16s$

Using Ratio Language

Notes and Guidance

Children will understand that a ratio shows the relationship between two values and can describe how one is related to another.

They will start by making simple comparisons between two different quantities. For example, they may compare the number of boys to girls in the class and write statements such as "for every one girl, there are two boys".

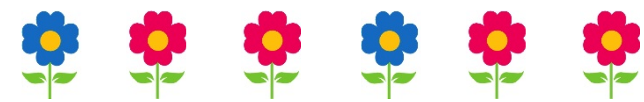
Mathematical Talk

How would your sentences change if there were 2 more blue flowers?
How would your sentences change if there were 10 more pink flowers?

Can you write a "for every…" sentence for the number of boys and girls in your class?

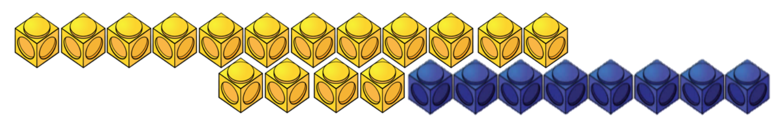
Varied Fluency

1. Complete the sentences.



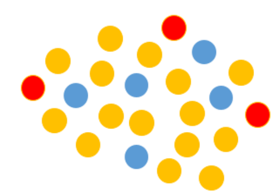
For every blue flower there are ………… pink flowers.
For every two blue flowers there are ………… pink flowers.

2. Rearrange the same number of cubes as there are in the diagram to help you complete the sentences.



For every ____ , there are ____ .
For every 8 , there are ____ .
For every 1 , there are ____ .

3. How many "for every…" sentences can you write to describe the counters?



Using Ratio Language

Reasoning and Problem Solving

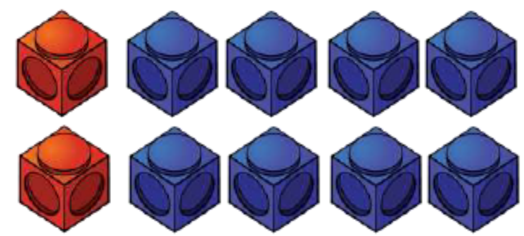
Tariq lays tiles in the following pattern:

If he has 16 blue tiles and 20 purple tiles can he continue his pattern without there being any tiles left over?

Explain why.

Possible response:
For every two blue tiles there are three purple tiles. If Tariq continues the pattern he will need 16 blue tiles and 24 purple tiles. He cannot continue the pattern without there being tiles left over.

True or False?



• For every red cube there are 8 blue cubes.	False
• For every 4 blue cubes there is 1 red cube.	True
• For every 3 red cubes there would be 12 blue cubes.	True
• For every 16 cubes, 4 would be red and 12 would be blue.	False
• For every 20 cubes, 4 would be red and 16 would be blue.	True

Ratio and Fractions

Notes and Guidance

Children are introduced to proportion by comparing a part to the whole.

They begin to see the link between to see the link between comparing quantities using ratio language (for every…..) and comparing quantities using fractions.

Mathematical Talk

How many apples are there compared to oranges?

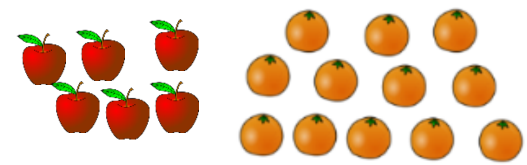
What fraction of the sweets are red, blue, orange?

Can I make a bar model to compare the quantities more efficiently?

What is the same and what is different about all the sentences you have written?

Varied Fluency

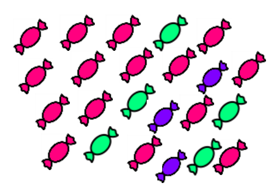
1. Complete the sentences to compare the apples and oranges.



For every 6 apples there are …… oranges.

$\frac{\square}{\square}$ of the fruit are apples, $\frac{\square}{\square}$ of the fruit are oranges.

2. Complete the sentences to compare the sweets.



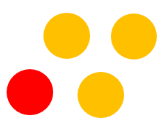
The number of pink sweets is ______ times the number of green sweets.
The number of pink sweets is ______ times the number of purple sweets.

$\frac{\square}{\square}$ of the sweets are pink, $\frac{\square}{\square}$ of the sweets are green.

For every 3 purple sweets there are ……. pink sweets.
For every 1 purple sweets there are …….. pink sweets.

Ratio and Fractions

Reasoning and Problem Solving

Fabio plants flowers in a flower bed. For every 2 red roses he plants 5 white roses. He says, $\frac{2}{5}$ of the roses are red. Is Fabio correct?	Fabio is incorrect because $\frac{2}{7}$ of the roses are red. He has mistaken a part with the whole.
Which is the odd one out? Explain your answer.   $\frac{1}{3}$	$\frac{1}{3}$ is the odd one out because the whole would be three parts whereas in the others the whole is four parts.

There are some red and green cubes in a bag. $\frac{2}{5}$ of the cubes are red. **True or False?** • For every 2 red cubes there are 3 green cubes. • For every 2 red cubes there are 5 green cubes. • For every 3 green cubes there are 2 red cubes. • For every 3 green cubes there are 5 red cubes. Explain your answers.	True False True False

Introducing the Ratio Symbol

Notes and Guidance

Children are introduced to the : notation and continue to link this with the language 'for every…, there are…'

Children understand that the notation relates to the order of parts. For example, 'For every 3 bananas there are 2 apples would be the same as 3 : 2 and for every 2 apples there are 3 bananas would be the same as 2 : 3

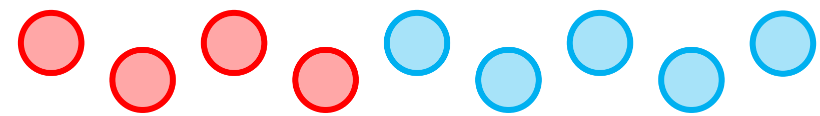
Mathematical Talk

What does the : symbol mean in the context of ratio?

Why is the order of the numbers important?

Varied Fluency

1. Complete.



The ratio of red counters to blue counters is $\square : \square$

The ratio of blue counters to red counters is $\square : \square$

2. Here are the ingredients for a smoothie.

Write down the ratio of:

- Bananas to strawberries
- Strawberries to bananas to blackberries
- Blackberries to strawberries to bananas
- Blackberries to strawberries

3. The ratio of red to green marbles it 3 : 7
Draw an image to represent the marbles.
What fraction of the marbles are red?
What fraction of the marbles are green?

Introducing the Ratio Symbol

Reasoning and Problem Solving

Tick the correct statements.  • There are two yellow tins for every three red tins. • There are two red tins for every three yellow tins. • The ratio of red tins to yellow tins is 2 : 3 • The ratio of yellow tins to red tins is 2 : 3 Explain which statements are incorrect and why.	The first and last statement are correct.	In a box there are some red, blue and green pens. The ratio of red pens to green pens is 3 : 5 For every 1 red pen there are two blue pens. Write down the ratio of red pens to blue pens to green pens.	3 : 6 : 5

Calculating Ratio

Notes and Guidance

Children build on their knowledge of ratios and begin to calculate ratios. They answer worded questions in the form of 'for every… there are …' and need to be able to find both a part and a whole.
They should be encouraged to draw bar models to represent their problems, and label clearly the information they have been given and what they want to calculate.

Mathematical Talk

Can we represent this ratio using a bar model?

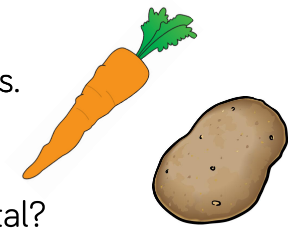
What does each part represent? What will each part be worth?

How can we share this quantity using the given ratio?

If we know what this part is worth, can we calculate the other parts?

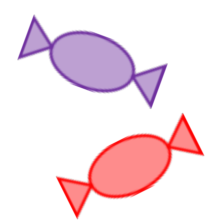
Varied Fluency

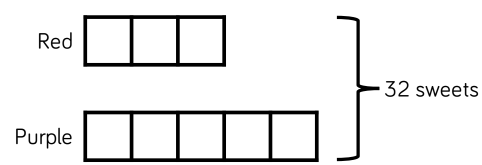
1. A farmer plants some crops in a field.
For every 12 carrots he plants 5 potatoes.
He plants 60 carrots in total.
How many potatoes did he plant?
How many vegetables did he plant in total?



2. Beth mixes 2 parts of red paint with 3 parts blue paint to make purple paint.
If she uses 12 parts blue paint, how much red paint did she use?

3. Emily has a packet of sweets.
For every 3 red sweets there are 5 purple sweets.
If there are 32 sweets in the packet in total, how many of each colour are there? You can use a bar model to help you.

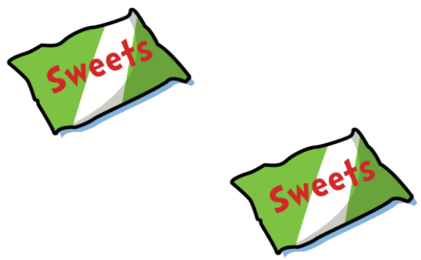




Calculating Ratio

Reasoning and Problem Solving

David has two packets of sweets.



In the first packet, for every one strawberry sweets there are two orange sweets.

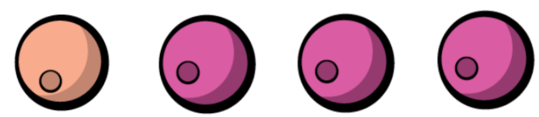
In the second packet, for every three orange sweets there are two strawberry.

Each packet contains 15 sweets in total.

Which packet has more strawberry sweets and by how many?

The first packet has 5 strawberry sweets and 10 orange sweets.
The second packet has 6 strawberry sweets and 9 orange sweets.
The second packet has 1 more strawberry sweet than the first packet.

Danielle is making some necklaces to sell. For every one pink bead, she uses three purple beads.



Each necklace has 32 beads in total.

The cost of the plain necklace is £2.80
The cost of a pink bead is 72 p
The cost of a purple bead is 65 p

How much does it cost to make one necklace?

Each necklace has 8 pink beads and 24 purple beads.

The cost of the pink beads is £5.76

The cost of the purple beads is £15.60

The cost of a necklace is £24.16

Using Scale Factors

Notes and Guidance

Once children are able to calculate ratio they can apply this knowledge to problems involving scale factors..

Children should be able to draw 2D shapes on a grid given a scale factor and be able to use vocabulary such as “shape A is three times as big as shape B”.

Mathematical Talk

What does enlargement mean?

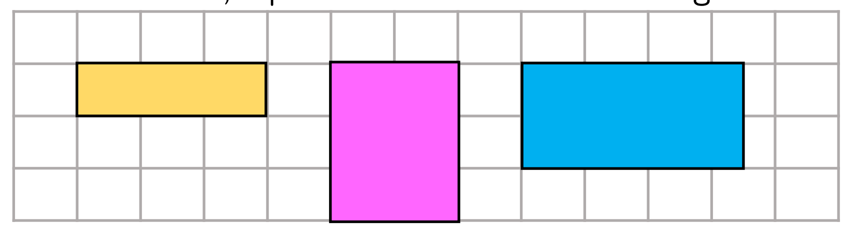
What does scale factor mean?

How much has the shape been increased by? How do you know? Can you prove it?

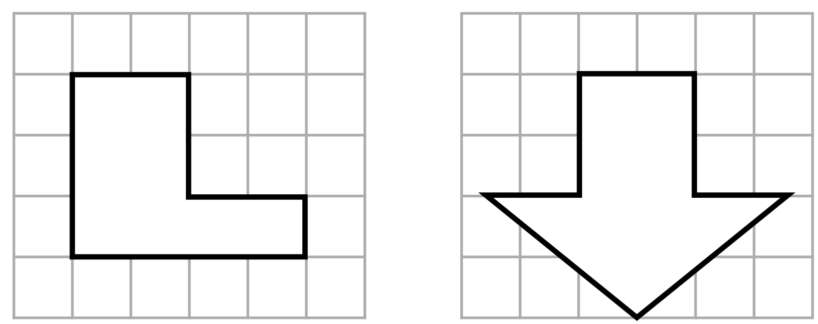
Have the angles changed size or not?

Varied Fluency

1 Copy these rectangles onto squared paper then draw them double the size, triple the size and 5 times as big.

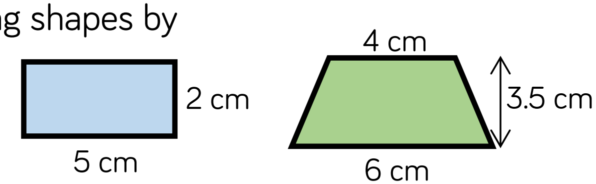


2 Copy these shapes onto squared paper then draw them twice as big and three times as big.



3 Enlarge the following shapes by

- Scale factor 2
- Scale factor 3
- Scale factor 4



Using Scale Factors

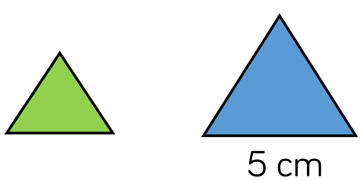
Reasoning and Problem Solving

Draw 3 rectangles with the same area where the length increases by the scale factor 2

Can you find more than one way of doing this?

Children could draw three rectangles with an area of 24 cm^2 where the length and width are 6 cm and 4 cm, 12 cm and 2 cm and 24 cm and 1 cm

Here are two equilateral triangles.
The blue triangle is three times larger than the green triangle.

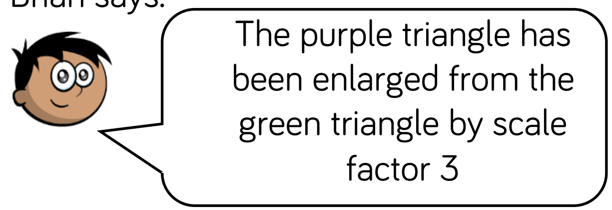


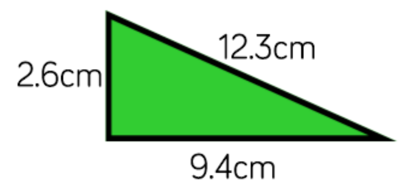
Find the perimeter of both triangles

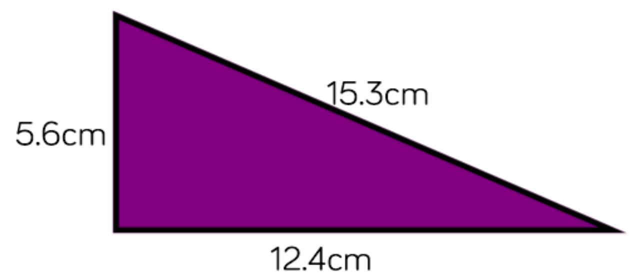
The blue triangle has a perimeter of 15 cm.

The green triangle has a perimeter of 5 cm

Brian says:







Do you agree?
Explain why.

Possible answer

I do not agree because Brian has increased the green shape by adding 3cm to each side, not increasing it by a scale factor of 3

Calculating Scale Factors

Notes and Guidance

Children find scale factors when given similar shapes. They continue to use scale factors to complete missing lengths.

Children use multiplication and division facts to accurately calculate missing information and scale factors.

Mathematical Talk

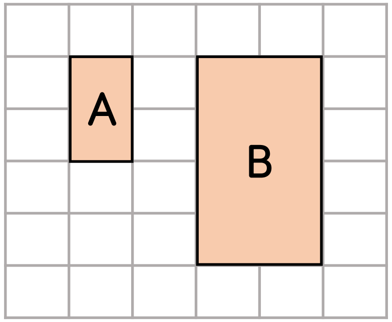
What do you notice about the length/width of each shape? Can you draw them?

How much larger/smaller is shape A compared to shape B?

What does a scale factor of 2 mean?

Varied Fluency

1. Complete the sentences to describe the shapes.



Shape B is __________ as big as shape A.

Shape A has been enlarged by scale factor ______ to make shape B.

2. The rectangles in the table are similar.
Fill in the missing lengths and widths and complete the sentences.

Rectangle	Length	Width
A	5cm	2cm
B		4cm
C	25cm	
D		18cm

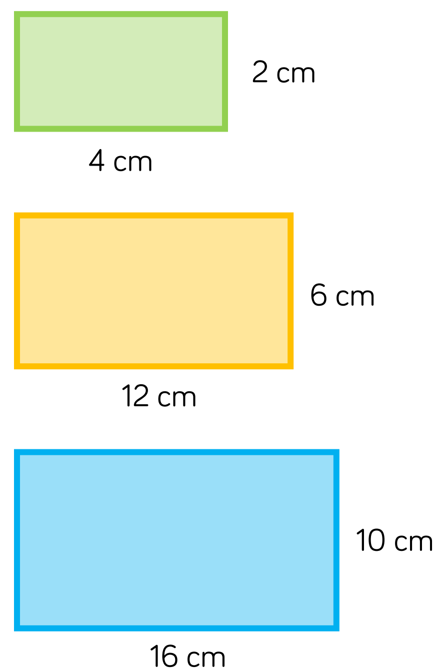
To enlarge A to B, use the scale factor ____
To enlarge A to C, use the scale factor ____
To enlarge A to D, use the scale factor ____
To enlarge B to D, use the scale factor ____

Calculating Scale Factors

Reasoning and Problem Solving

One rectangle has a perimeter of 16 cm. An enlarged version of this rectangle has a perimeter of 24cm. The length of the smaller rectangle is 6cm. Draw both rectangles.	Smaller rectangle: length – 6 cm width – 2 cm Larger rectangle: length – 9 cm width – 3 cm Scale factor: 1.5
Always, sometimes, never. To enlarge a shape you just need to do the same thing to each of the sides.	Sometimes. This only works when we are multiplying or dividing the lengths of the sides, it does not work with addition and subtraction etc.

Tom says that each of these rectangles is an enlarged version of one of the others.



Do you agree?
Explain your answer.

Tom is wrong. The orange rectangle is an enlarged version of the green with scale factor 3, but the blue rectangle is not similar because the same amount has been **added** to the sides and they should be multiplied or divided to be enlarged.

Ratio and Proportion Problems

Notes and Guidance

Children will draw together all their experiences of ratio and proportion to answer a variety of problems which will include a range of contextualised problems.

Mathematical Talk

Which model can help us visualise this problem?

Can we represent this ratio using a bar model?

What does each part represent?

What is the same about the ratios?

What is different about them?

Varied Fluency

1. The recipe to make soup for 6 people is given. How much of each ingredient will be needed to make the soup for:
   - 3 people
   - 9 people
   - 1 person

Recipe for 6 people

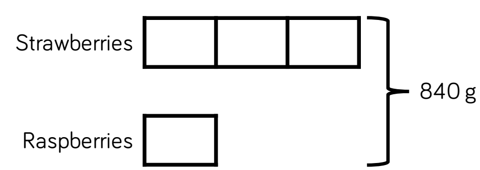
- 1 onion
- 60g butter
- 2 tbsp plain flour
- 2.4 litres stock
- 480ml tomato juice

2. Find the cost of one pen from each shop.

ASDI	TESCU
4 pens £2.88	7 pens £4.83

Which is better value?

3. A smoothie contains three times as many strawberries as raspberries. The combined weight of the strawberries and raspberries is 840 g. What weight of strawberries is needed?



Ratio and Proportion Problems

Reasoning and Problem Solving

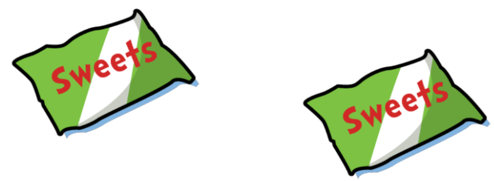
Here is the recipe for making flapjacks.

<u>Flapjacks</u>
Serves: 10
120 g butter
100 g dark brown soft sugar
4 tablespoons golden syrup
250 g rolled oats
40 g sultanas or raisins

Jonathan has 180 g butter.
What is the largest number of flapjacks he can make?
How much of everything else will he need?

He has enough butter to make 15 flapjacks.
He will need 150 g dark brown soft sugar, 6 tablespoons golden syrup, 375 g rolled oats and 60 g sultanas or raisins.

Jodie has two packets of sweets.



In the first packet, for every 2 strawberry sweets there are 3 orange.

In the second packet, for one strawberry sweet, there are three orange.

Each packet has the same number of sweets.

The second packet contains 15 orange sweets.

How many strawberry sweets are in the first packet?

Second packet:

15 orange

5 strawberry

So there are 20 sweets in each packet.

First packet:

8 strawberry

12 orange

The first packet contains 8 strawberry sweets.