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# **Small Steps Guidance and Examples**

**Block 3: Multiplication & Division** 



#### Year 5/6 | Autumn Term | Teaching Guidance

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#### Year 5 | Autumn Term | Teaching Guidance

Week 6 to 8 – Number: Multiplication and Division

#### **Multiples**

#### Notes and Guidance

Building on their times tables knowledge, children will find multiples of whole numbers. Children build multiples of a number using concrete and pictorial representations e.g. in an array.

#### Varied Fluency

Circle the multiples of 5.

25 32 54 40 175 3000

What do you notice about the multiples of 5?



Write all the multiples of 4 between 20 and 80.

3 Roll 2 die (1-6), multiply the numbers. What is the number a multiple of? Is it a multiple of more than one number?

How many different numbers can you make multiples of? Can you make multiples of all numbers up to 10? Can you make multiples of all numbers up to 20? Use a table to show your results. Multiply the numbers you roll to complete the table. An example is shown below

#### Mathematical Talk

- What do you notice about the multiples of 2? What is the same about them, what is different?
- Look at multiples of other numbers; is there a rule that links them?

# **Multiples**

# **Reasoning and Problem Solving**

Use the digits 0 – 9. Choose 2 digits. Multiply them together.

What is your number a multiple of?

Is it a multiple of more than one number?

Can you find all the numbers you could make?

Use the table below to help.

	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

Always- Two even
together are all
Never- Two odd numbers multiplied together are always a multiple of an odd number. You cannot make a multiple of an even
nornoer.
Clare is 21 years old,

#### Year 6 | Autumn Term | Teaching Guidance

Week 6 to 8 – Number: Multiplication & Division

#### **Common Multiples**

#### Notes and Guidance

Building on knowledge of multiples, children find common multiples of numbers. They should continue to use a visual representation to support their thinking. They also use more abstract methods to calculate the multiples and use numbers outside of times table facts.

#### Mathematical Talk

Are the lowest common multiples of a pair of numbers always the product of them? Can you think of any strategies to work out the lowest common multiples of different numbers? When do numbers have common multiples that are lower than their product?

#### Varied Fluency

On a 100 square, shade the first 5 multiples of 7 and then the first 8 multiples of 5 What do you notice? Choose 2 other times tables which you think will have more than 3 common multiples.



#### List 5 common multiples of 4 and 3

3

Jim and Nancy play football at the same local football pitches. Jim has plays once every 4 days and Nancy plays once every 6 days. In a fortnight, how many times will they play football on the same day?

# **Common Multiples**

Work out the headings for the Venn diagram.	Headings: Multiples of 4 Multiples of 6 144 is a multiple of 6 and 8	lings: ples of 4 ples of 6Nancy is double her sister's age.They are both older than 20 and younger than 50Their ages are both multiples of 7Work out their ages.		Nancy: 42 Nancy's sister: 21
Can you think of a multiple of 6 and 8 that is a square number?			<ul><li>Train starts running from Leeds to York at 7am.</li><li>The last trains leaves at midnight.</li><li>Platform 1 has a train leaving from it every 12 minutes.</li><li>Platform 2 has one leaving from it every 5 minutes.</li><li>How many times in the day would there be a train leaving from both platforms at the same time?</li></ul>	Platform 1 and 2 will have a train leaving at the same time once every hour at o'clock. Therefore there will be 18 times from 7am to midnight when a train will leave at both platform 1 and 2

#### Year 5 | Autumn Term | Teaching Guidance

# Multiplying by 10, 100 & 1000

#### Notes and Guidance

Children recap multiplying by 10 and 100 before moving on to multiplying by 1000. They look at numbers in a place value grid and discuss how many places to the left digits move when you multiply by different multiples of 10.

Mathematical Talk

- Which direction do the digits move when you multiply by 10, 100 or 1000?
- How many places do you move to the left?
- When we have an empty place value column to the right of our digits what number do we use as a place holder?
- Can you use multiplying by 100 to help you multiply by 1000? Explain why.

#### Varied Fluency

Make the number 234 on the place value grid using counters.

HTh	TTh	Th	Н	Т	0
			•		

When I multiply my number by 10, where will I move my counters?

Remember when we multiply by 10, 100, 1000, we move the digits to the left and use zero as a place holder.



Complete the following questions using counters and a place value grid.

234 × 100 =	324 × 100 =
100 × 36 =	1,000 × 207 =
45,020 × 10 =	= 3,456 × 1,000



# Multiplying by 10, 100 & 1000

Rosie has £300 in her bank account. Louis has 100 times more than Rosie in his bank account. How much more money does Louis have than Rosie?	Rosie has £300 Louis has £30,000 Louis has £27,700 more than Rosie.	<ul><li>Jack is thinking of a 3-digit number.</li><li>When he multiplies his number by 100, the ten thousands and hundreds digit are the same.</li><li>The sum of the digits is 10.</li><li>What number could Jack be thinking of?</li></ul>	181, 262, 343, 424, 505
Emily has £1020 in her bank account and Philip has £120 in his bank account. Emily says, 'I have ten times more money than you.' Is Emily correct? Explain your reasoning.	No. Emily would have £1200 if this was the case.		

#### **Mental Calculations**

#### Notes and Guidance

We have included this small step separately to ensure that teachers give emphasis to this important skill. Discussions around efficient mental calculations and sensible estimations need to run through all steps.

Sometimes children are too quick to move to computational methods, when changing the order leads to quick mental methods and solutions.

#### Mathematical Talk

Is there an easy and quick way to do this?

Can you use known facts to answer the problem?

Can you use rounding?

Does the solution need an exact answer?

How does knowing the approximate answer help with the calculation?

# Varied Fluency

How could you change the order of these calculations to be able to perform them mentally?

50 × 16 × 2 = 30 × 12 × 2 = 25 × 17 × 4 =

2

Jamie buys a t shirt for £9.99, socks for £1.49 and a belt for £8.99 He was charged £23.47

How could he quickly check if he was overcharged?

3 A ↓ C

What do you estimate that B represents when: A = 0 and C = 1,000 A = 30 and C = 150 A = -7 and C = 17 A = 0 and C = 5,000A = 1,000 and C = 100,000

# **Mental Calculations**



#### Year 5 | Autumn Term | Teaching Guidance

#### Dividing by 10, 100 & 1000

#### Notes and Guidance

Children look at dividing by 10, 100 and 1000 using a place value chart. They use counters and digits to learn that the digits move to the right when dividing by powers of ten.

#### Mathematical Talk

- What happens to the digits?
- How are dividing by 10, 100 and 1,000 related to each other?
- How are dividing by 10, 100 and 1,000 linked to multiplying by 10, 100 and 1,000?

What does 'inverse' mean?

# Varied Fluency

TTh

н

Т

0

Th

What number is represented in the place value grid? Divide the number by 100. Which direction do the counters move? How many columns do they move? What number do we have now?

HTh

Complete the following using the place value grid. Divide 460 by 10 Divide 5,300 by 100 Divide 62,000 by 1000

Divide the following numbers by 10, 100 and 1000 80,000 300,000 547,000



Calculate 45,000 ÷ 10 ÷ 10 How else could you write this?

# Dividing by 10, 100 & 1000

David has £357,000 in his bank. He divides the amount by 1,000 and takes that much money out of the bank. Using the money he has taken out he spends £269 on furniture for his new house. How much money does David have left from the money he took out? Show your workings out.	357,00 ÷ 1,000 = 357 If you subtract £269, he is left with £88	Here are the answers to some problems: 5700 405 397 6,203 Can you write at least two questions for each answer involving dividing by 10, 100 or 1000?	Possible solutions could be: 3970 ÷10 = 397 57,000 ÷10 = 5,700 397,000 ÷ 1000 = 397 40,500 ÷ 100 = 405 620,300 ÷ 100 = 6,203
		Match the calculation to the answer:	The missing answer
Apples weigh about 160g each. How many apples would you expect to get in a 2kg bag? Explain your reasoning.	Children need to be able to use knowledge of equivalent measures to convert 2kg to 2,000g. There are approximately 12 apples.	$64, 640, 6,400$ $64,000 \div 10$ $640 \div 10$ $640,000 \div 1000$ $6,400 \div 100$ $6400 \div 10$ $64,000 \div 1000$ $64,000 \div 100$ $640,000 \div 100$ How do you know? Do any of the calculations have the same answers?Is there an answer missed out? Explain what you have found.	is 64,000. Children could use place value grids to demonstrate the digits moving columns.

#### Year 5 | Autumn Term | Teaching Guidance

# Multiples of 10, 100 & 1000

#### Notes and Guidance

Children have been taught how to multiply and divide by 10, 100 and 1000.

They now use knowledge of other multiples to calculate related questions.

# Mathematical Talk

If we are multiplying by 20, can we break it down into two steps and use our knowledge of multiplying by 10?

How does using multiplication and division as inverses help us use known facts?

# Varied Fluency



	INEtHOU ∠
24 × 10 × 2	24 × 2 × 10
= 240 × 2	$= 48 \times 10$
= 480	= 480

What is the same about the methods, what is different?

3

Use the division diagram to help solve the calculations.  $7,200 \div 200 = 36$ 

7.200 ÷ 100 72 ÷2 36

$$3,600 \div 200 =$$

$$18,000 \div 200 =$$

$$5,400 \div = 27$$

$$= 6,600 \div 200$$

# Multiples of 10, 100 & 1000



#### Year 6 | Autumn Term | Teaching Guidance

#### **Reason from Known Facts**

#### Notes and Guidance

Pupils should be able to use their understanding of known facts from one calculation to work out the answer of another similar calculation without starting afresh.

They should use reasoning and apply their knowledge of commutativity and inverse.

# Mathematical Talk

What is the inverse?

When do you use the inverse?

How can we use multiplication/division facts to help us answer similar questions?

# Varied Fluency

$$70 \div = 3.5$$
  

$$70 \div = 7$$
  

$$- \div 2 = 35$$
  

$$- \times 3.5 = 7$$
  

$$3.5 \times 20 = -$$
  

$$70 \div = 3.5$$

Make a similar set of calculations for  $90 \div 2 = 45$ 



 $5138 \div 14 = 367$ Use this to work out 15 × 367



 $14 \times 8 = 112$ Use this to work out:



# **Reason from Known Facts**



#### Year 5 | Spring Term | Teaching Guidance

# Multiply up to 4-digits by 1-digit

#### **Notes and Guidance**

Children build on previous steps to represent a four-digit number multiplied by a one-digit number with concrete manipulatives.

Teachers should be aware of misconceptions arising from 0 in the hundreds, tens or ones column.

Children then move on to explore multiplication with exchange in first one column and then more than one column.

#### Mathematical Talk

Why is it important to set out using columns?

Explain the value of each digit in your calculation.

What happens when there is a 0 in the ones column, tens column or hundreds column?

What do we do if there are ten counters in a column?

# Varied Fluency



	Th	Н	т	0
	1	0	2	3
X				3





Write the multiplication calculation represented and find the answer.



Remember if there are ten or more counters in a column, make an exchange.



Sam earns £1,325 per week. How much would he earn in 4 weeks?

TH	н	Т	0					
<b>•••</b>	100 100 100	10 10	<b>00000</b>		Th	Н	Т	0
	100 100 100	10 10			1	3	2	5
	100 100 100	10 10		×				4
1000	100 100 100	10 10	<b>0 0 0 0 0</b>	_				

# Year 5 | Spring Term

# Multiply up to 4-digits by 1-digit

# **Reasoning and Problem Solving**

Megan worked out the answer to 1,432 $\times$ 4 Here is her answer:	
ThHTO1432 $\times$ 41612416128Can you explain what Megan has done wrong?	8 Megan has not exchanged when she has got 9 or more in the tens and hundreds columns.
Can you create a calculation where the answer involves:	Possible answers:
a) An exchange in the ones column only	a) 2,317 × 3 =
b) An exchange in the tens column only.	B )2,152 × 4 =
c) An exchange in the hundreds column only.	c) 1,632 × 3 =
d) More than one exchange.	d) 4,023 × 5 =
e) A zero in one of the columns.	e) 3,152 × 4 =

Can you work out the missing numbers using the clues?



Answer:

#### $2,345 \times 5 = 11,725$

The 4 digits being multiplied by 6 are consecutive numbers. The first 2 digits of the answer are the same. The 4th and 5th digits in the answer add to make the 3rd.

#### Year 5 | Spring Term | Teaching Guidance

# Multiply 2-digits (1)

#### Notes and Guidance

Children use base ten to represent the area model of multiplication. The base ten will enable children to see the size and scale linked to multiplying.

Children will then move on to representing this more abstractly with place value counters alongside numbers.

#### Mathematical Talk

How can we represent this calculation? What are we multiplying?

What is happening in step 2? How have we partitioned our numbers? How does this help when multiplying?

Where can we see  $20 \times 20$ ? What does the 40 represent?

What's the same and what is different between the three representations (base ten, place value, grid)?

# Varied Fluency

Joshua uses the base ten to solve  $23 \times 22 =$ 



Can you use the base ten to solve the following calculations?

$$32 \times 24 = 25 \times 31 = 34 \times 23 =$$



	44 × 32 =			44 × 32	-	44 × 32 =			
	40	4		40	4		40	4	1200
30			30	0000	0000	30	1200	120	1200
				0000		2	80	8	8
2			2	8888	8888				1400
				40	4				
			30	1200	120				
			2	80	8				



 $45 \times 42 =$ 

Can you use place value counters and the grid to solve the following calculations?

 $52 \times 24 =$ 

 $34 \times 43 =$ 

# Year 5 | Spring Term

# Multiply 2-digits (1)

#### Reasoning and Problem Solving

#### Tabby says:



If I multiply  $23 \times 57$ , I can just do these calculations,  $20 \times 50$ and  $3 \times 7$  and then add the totals.

Do you agree? Convince me!

Ryan hasn't finished his calculation. Can you complete the missing information and record his calculation with an



Possible response: I disagree because if you partition each number you have 20 + 3 and 50 +7. You will have to do  $20 \times 50$  and  $20 \times 7$  add  $3 \times 50$ and  $3 \times 7$ .

Possible response: Ryan needs 8 more hundreds.  $40 \times 40$ = 1,600 and he only has 800.

His calculation is  $42 \times 46 = 1,932$ 

Farmer Joe has a field that measures 53 m long by 25 m wide.

Farmer Sally has a field that measures 52 m long by 24 m wide.

Max thinks they will have the same area because the numbers have only changed by one digit each.

Do you agree? Prove it!



Children may prove this with concrete or pictorial representations.

For example: I disagree because...



# Multiply 2-digits (2)

#### Notes and Guidance

Children will move on from the area method and work towards more formal multiplication.

They will start by exploring the role of the zero in the calculation and why it's important. Children should understand what is happening through each step of the calculation process.

Mathematical Talk

- What is the same about the grid, expanded and compact methods?
- What is the different about the grid, expanded and compact methods?
- Why is the zero important?
- What numbers are being multiplied in the first line and the second line?
- What do we do with an exchange? What happens if the exchange is more than 1?
- If we know what 38  $\times$ 12 is, how else could we work out 39  $\times$  12?

# Varied Fluency

What is the same about the methods below? What is different?

Compact	Expand		Grid	
23 × 14	23 × 14	×	20	3
92 (23 × 4) 230 (23 × 10)	$ \begin{array}{c} 12 & (3 \times 4) \\ 80 & (20 \times 4) \\ 70 & (7 \times 10) \end{array} $	10	200	30
	$\frac{200}{322}$ (20 × 10)	4	80	12

Where can you see  $20 \times 10$  in the expanded and the grid calculations? Why can't we see this in the compact method? Use the methods above to calculate:

> $34 \times 26$  $58 \times 15$  $72 \times 35$



2

3

Complete the following to solve the calculation:



# Year 5 | Spring Term

# Multiply 2-digits (2)



#### Year 5 | Spring Term | Teaching Guidance

# Multiply 3-digits by 2-digits

#### Notes and Guidance

Children will extend their multiplication skills to multiplying three-digit numbers by two-digit numbers.

They will use multiplication to find area and solve multi-step problems.

Mathematical Talk

Why is the zero important?

What numbers are being multiplied in the first line and the second line?

What do we do with an exchange? What happens if there is an exchange in the last column we multiply in?

#### 



A playground is 128 yards by 73 yards. Calculate the area of the playground.



# Year 5 | Spring Term

# Multiply 3-digits by 2-digits

#### **Reasoning and Problem Solving**

Tiffany has spilt paint on her maths homework, can you work out the digits that have been covered by paint?



Pencils come in boxes of 64 and a school bought 270 boxes. The school ordered 720 boxes of handwriting pens and there are 46 pens per box. How many more pens are there than pencils?







#### Answer: 270 × 64 = 17, 280 720 × 46 = 33, 120 33, 120-17, 280 =15, 840

Here are exam work:	ples of Casey's maths
a)	b)
987	324
<u>× 76</u>	× 78
5 <sub>5</sub> 9 <sub>4</sub> 22	$15_{3}92(324 \times 8)$
<sup>+</sup> 6 <sub>6</sub> 9₄0 9	$\frac{1}{2}$ 2 <sub>2</sub> 6 8 0 (324 × 70)
1,28,31	

Can you explain the mistakes she's made? Correct each calculation.

#### Answer:

A) Casey has forgotten to write the zero to show that she is multiplying by 10. 987 $\times 76$ 5922

B) Casey has not exchanged into the thousand or ten thousand columns correctly

	324
	× 78
	2 <sub>1</sub> 5 <sub>3</sub> 9 2
+	$2_1 2_2 6 8 0$
	25272

#### Year 5 | Spring Term | Teaching Guidance

### Multiply 4-digits by 2-digits

#### Notes and Guidance

Children will build upon previous learning of multiplying a 3 digit number by a 2 digit number and apply this to 4 digit numbers.

It is important that children remember the steps to take when using this multiplication method.

Mathematical Talk

What steps do we need to go through when using this multiplication method?

Look at the numbers in each question, can they help you estimate which answer will be the largest?

#### Varied Fluency



× +<sup>1</sup> 6

Use the method shown to complete.

3250 26	×
9500 × 5000 ×	+



Calculate: 9,708 × 38 = 7,132 × 21 = 3,282 × 32 =



Order the following from smallest to greatest.

4.458 × 56 4,523 × 54  $4.535 \times 55$ 

# Year 5 | Spring Term

# Multiply 4-digits by 2-digits

#### **Reasoning and Problem Solving**



Charlotte owns an amusement park. Which ticket type made more money?

Shown how you know this.

Ticket type	Price	Visitors
Mon-Thurs	£23	3,456
Fri-Sun	£28	2,827





Monday to Thursday makes more money because 79,488-79,156=332 There is a £332 difference.

#### Year 6 | Autumn Term | Teaching Guidance

# Multiply 4-digits by 2-digits

#### Notes and Guidance

Children consolidate their knowledge of column multiplication.

They use these skills to solve multi step problems in a range of contexts.

#### Mathematical Talk

What is important to remember as we begin multiplying by the tens number?

How would you draw the calculation?

- Can the inverse operation be used?
- Is there a different strategy that you could use?

# Varied Fluency



Calculate

	4267		3046
×	34	×	73

#### 5734 × 26 =



Lauren made cookies for a bake sale. She made 345 cookies. The recipes stated that she should have 17 chocolate chips in each cookie. How many chocolate chips will there be altogether?

#### 3

Work out the missing number.



# Multiply 4-digits by 2-digits

#### **Reasoning and Problem Solving**

#### True or false.

- a) 5,463 × 18 is the same as 18 × 5,463
- b) I can find the answer to 1,100 × 28 by using 1,100 × 30 and taking away two lots of 1,100

c)  $70 \div 10 = 700 \div 100$ 

- a) True because multiplication is commutative so the calculation can be done in any order
- b) True because they both show 28 lots of 1,100
- c) True because both numbers have been made 10 times bigger



Place the digits in the boxes to make the largest product.



8432 × 75

#### Year 5 | Spring Term | Teaching Guidance

# Divide up to 4-digits (1)

#### Notes and Guidance

Children build on their knowledge from Year 4 of dividing three digits by one-digit. Following on from this they divide up to four-digit numbers by a one-digit number.

They use place value counters to group their number and develop their understanding of the short division method.

Mathematical Talk

How many groups of \_\_\_\_\_ thousands are there in \_\_\_\_\_ thousand? How many groups of \_\_\_\_\_ hundreds are there in \_\_\_\_\_ hundreds? How many groups of \_\_\_\_\_ tens are there in \_\_\_\_\_ tens? How many groups of \_\_\_\_\_ ones are there in \_\_\_\_\_ ones?

Do I need to solve both calculations to compare the divisions?

# Varied Fluency

Here is a method to solve 4,892 divided by 4 using place value counters and short division.



Use this method to solve the following questions.

6,610 ÷ 5 2,472 ÷ 3 9,360 ÷ 4



3

Mr Porter has saved £8,934 pounds. He shares it between his three grandchildren. How much does each grandchild receive?

Use < > or = to compare the statements:

# Year 5 | Spring Term

# Divide up to 4-digits (1)

#### Reasoning and Problem Solving

Sam is working out 2,240 divided by 7. He says you can't do it because 7 is larger than all of the digits in the number.

Do you agree with Sam? Explain your answer. Sam is incorrect because you can exchange between columns. When 2 thousand doesn't group into 7's, you can exchange 2 thousands for 20 hundreds. This will then divide by 7. 2,240 divided by 7 equals 320. Spot the mistake.

Explain and correct the working.



There is no exchanging within the calculation when there are remainders from a division. In the hundreds the hundred should have been exchanged for ten tens. There should have been two tens exchanged into the ones. The final answer should have been 3,138

#### Year 6 | Autumn Term | Teaching Guidance

Week 6 to 8 – Number: Multiplication & Division

#### **Short Division**

#### Notes and Guidance

Children build on their understanding of dividing up to 4 digits by 1 digit by now dividing by up to 2 digits. They use the short division and focus on division as grouping.

Teachers may encourage children to list the multiples of the number to help them solve the division more easily.

#### Mathematical Talk

What is different between dividing by 1 digit and 2 digits? If the number does not divide into the ones, what do we do?

Do we need to round our remainders up or down? Why does the context affect whether we round up or down?

#### Varied Fluency



List the multiples of the number to help you calculate.

- 2
- A limousine company allows 14 people per limousine. How many limousines need to be hired for 230 people?
- 3

Year 6 have 2,356 pencil crayons for the year. They put them in bundles with 12 in each bundle. How many complete bundles can be made?

# **Short Division**



#### Year 5 | Spring Term | Teaching Guidance

# Divide up to 4-digits (2)

#### Notes and Guidance

Children continue to use place value counters to group their number and develop their understanding of the short division method.

They start to focus on remainders and build on their learning from Year 4 to understand remainders in a context.

They do not represent remainders as a fraction at this point.

# Mathematical Talk

- If I can't make a group in this column, what do I do?
- What happens if we can group the one equally?
- In this number story, what does this remainder mean?
- When would we round the remainder up or down?
- What context would we just focus on the remainder?

# Varied Fluency





Use this method to solve the following questions.

$$6,613 \div 5 = 2,471 \div 3 = 9,363 \div 4 = 0$$

(2)

3

Muffins are packed in trays of 6 in a factory. In one day a factory makes 5,623 muffins.

How many trays do they need per day?

How many full trays do they have at the end of the day?

For the calculation, 8, 035  $\div$  4, can you:

Write a number story where you have to round the remainder up. Write a number story where you have to round the remainder down.

Write a number story where you have to find the remainder.

# Year 5 | Spring Term

# Divide up to 4-digits (2)

#### **Reasoning and Problem Solving**

I am thinking of a three-digit number. When it is divided by 9, the remainder is 3. When it is divided by 2, the remainder is 1. When it is divided by 5, the remainder is 4. What is my number?



If there is a remainder of 4 when dividing by 5, the ones digit must be 4 or 9. If there is a remainder of 1 when dividing by 2, the number must be odd so the ones digit must be 9. In a multiple of 9, the digits always add up to a multiple of 9, this will help me to check if my number will have a remainder of 3 when divided by 9. Possible answers: 129, 219, 309, 399,

489, 579, 669, 759,

765 ÷ 4 = 191 remainder 1	
876 ÷ 5 = 175 remainder 1	This does not always have the
Does a three-digit number descending in digits divided by the next descending digit always have remainder 1?	$543 \div 2 = 271 \text{ r1}$
Prove your answer.	However: $987 \div 6 = 164 \text{ r}3$
	654 ÷ 3 = 218

#### **Division using Factors**

#### Notes and Guidance

Children need to use their number sense, specifically their knowledge of factors to be able to see relationships between the divisor and dividend. Beginning with multiples of 10 and moving on will allow the children to see the relationship before progressing forward.

Mathematical Talk

What is a factor?

How does using factor pairs help us to answer division questions?

Do you notice any patterns?

Does using factor pairs always work?

- Is there more than one way to solve a calculation using factor pairs?
- What methods can be used to check your working out?

#### Varied Fluency

780 ÷ 20 = 39 is the same as 780 ÷ 10 = 78 then 78 ÷ 2 = 39

What do you notice?

Use the same method to solve  $480\div60$ 



Use factors to help you to answer

4,320 ÷ 15



Eggs are put into boxes holding a dozen. A farmer wants to put 648 eggs into boxes. How many boxes will he have filled?



14

# **Division using Factors**

Divide 1,248 by • 48 • 24 • 12 What did you do each time? Explain your strategy.	$1,248 \div 48 = 26$ $1,248 \div 24 = 52$ $1,248 \div 12 = 104$ I used factor pairs to complete the first question e.g. I divided 1,248 by 12 then divided the answer by 4 Because 24 is half of 48, I doubled 26 to get 52 I repeated this with 12 to get 104	Class 6 are solving 7,848 ÷ 24 The children decide which factor pairs to use between: • 2 and 12 • 4 and 6 • 10 and 14 Which will not give them the correct answer? Why?	10 and 14 will not give them the correct answer because 10 and 1 are not factors of 24	
Ivan To work out 4,320 ÷ 15 I will first divide 4,320 by 5 then divide the answer by 10 Is Ivan correct? Explain why.	Ivan is incorrect. He has partitioned 15 when he should have used factor pairs e.g. 5 and 3 The answer is 288			

# Long Division (1)

#### Notes and Guidance

Children are introduced to long division as a different method of dividing by a 2-digit number. They divide 3digit numbers by a 2-digit number without remainders moving from a more expanded method with multiples shown to the more formal long division method.

#### Mathematical Talk

How can we use our multiples to help us divide by a 2digit number?

Why are we subtracting the totals from the beginning number (seeing division as repeated subtraction)?

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided)

### Varied Fluency

1					3	6		Multiples to help
	1	2		4	3	2		$12 \times 1 = 12$
			_	3	6	0	(×30)	$12 \times 2 = 24$
					7	2		$12 \times 5 = 60$
			_		7	2	(×6)	$12 \times 10 = 120$
						0	)	

Solve the following divisions using the method above. Write out your multiples that may help you.

1

 $765 \div 17 = 450 \div 15 = 702 \div 18 =$ 

Use the long division method to solve the following calculations. One has been done for you as an example.

 $836 \div 11 =$ 798 ÷ 14 = 608 ÷ 19 =

2

#### Week 6 to 8 – Number: Multiplication & Division

# Long Division (1)

# Reasoning and Problem Solving

Which calculation could be the odd one out below?

- 512 ÷ 16 =
- 672 ÷ 21=
- 928 ÷ 29 =
- 792 ÷ 24 =

Explain why.

512 ÷ 16 = 32	
672 ÷ 21 = 32	
928 ÷ 29 = 32	
792 ÷ 24 = 33	

Possible answers: 928 ÷ 29 is the odd one out because it is the only 3-digit number without a 2 in the ones column.

 $792 \div 24$  is the odd one out because it is does not have the answer 32 Explain the mistake

Instead of writing 10 lots of 16 as 160 they have written 10 lots of 16 as 106 This is therefore the mistake in the calculation.

#### Year 6 | Autumn Term | Teaching Guidance

Week 6 to 8 – Number: Multiplication & Division

#### Long Division (2)

#### Notes and Guidance

Building on using long division with 3 digit numbers, children divide four digit numbers by 2 digits using the long division method.

They use their knowledge of multiples and multiplying and dividing by 10 and 100 to calculate more efficiently.

#### Mathematical Talk

How can we use our multiples to help us divide by a 2-digit number?

Why are we subtracting the totals from the beginning number (seeing division as repeated subtraction)?

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided)

# Varied Fluency

	0 4 8 9		Here is a division
1 5	7 3 3 5		method. Solve the divisions using this
-	6 0 0 0	(× 400)	method.
_	1 3 3 5	(× 80)	2,208 ÷ 16 =
_	1 2 0 0	(1100)	1.755 ÷ 45 =
	1 3 5		.,
-	1 3 5	(× 9)	= <b>1</b> ,536 ÷ 16
	0		



There are 2,028 footballers in tournament. Each team has 11 players and 2 substitutes. How many teams are in the tournament?



= 23

# Long Division (2)

#### **Reasoning and Problem Solving**

Which question is easier and which is harder?

- 1,950 ÷ 13 =
- 1,950 ÷ 15 =

Explain why.

1,950 ÷ 13 is harder because 13 is a prime number and therefore cannot be split into factors and divided in smaller parts.

6,823 ÷ 19 = 359 r2 8,259 ÷ = 359 r2 Find the value of

# Long Division (3)

#### Notes and Guidance

Children now divide using long division where their answers have remainders. After dividing, they check that their remainder is smaller than their divisor.

Children start to understand when rounding is appropriate to use for interpreting the remainder and when the context means that this is not applicable.

#### Mathematical Talk

How can we use our multiples to help us divide?

What happens if we cannot divide our ones exactly by our divisor? How do we show what we have left over?

Why are we subtracting the totals from the starting amount (seeing division as repeated subtraction)?

Does the remainder need to be rounded up or down?

#### Varied Fluency



2

Elijah uses this method to calculate 372 divided by 15. He has used his knowledge of multiples to help.

	2	4 r 1 2	1 × 15 = 15
15	3 7 2		$2 \times 15 = 30$
	 - 3 0 0	(× 20)	3 × 15 = 45
	7 2	,	4 × 15 = 60 5 × 15 = 75
	- 6.0	- (× 4)	$10 \times 15 = 150$

1 2

Solve the following calculations using Elijah's method. Show the multiples that you need to use to help you.

271 ÷ 17 = 623 ÷ 21 = 842 ÷ 32 =

A school needs to buy 380 biscuits to pass around at parents' evening. They come in packets of 12. How many packets with the school need to buy?

# Long Division (3)



# Long Division (4)

#### Notes and Guidance

Children now divide four-digit numbers using long division where their answers have remainders. After dividing, they check that their remainder is smaller than their divisor.

Children start to understand when rounding is appropriate to use for interpreting the remainder and when the context means that this is not applicable.

Mathematical Talk

How can we use our multiples to help us divide?

What happens if we cannot divide our ones exactly by our divisor?

How do we show what we have left over?

Why are we subtracting the totals from the starting amount (seeing division as repeated subtraction)?

Does the remainder need to be rounded up or down?

#### Varied Fluency

Simon used this method to calculate 1426 divided by 13. He wrote down his multiples key facts to help him work out the answer.

				1	. 0	9	r	9
1	3		1	4	2	6		-
		_	1	3	0	0		(×100)
			0	1	2	6		-
		-		1	1	7		(× 9)
				0	) ()	9		-

Using Simon's method answer the following:  $2,637 \div 16 =$  $4,231 \div 22 =$  $4,203 \div 18 =$ 



There are 7,849 people going to a concert. Each coach holds 64 people. How many coaches are needed to transport all the people?

# Long Division (4)

Class 6 are completing this calculation $3,636 \div 12$	Violet is incorrect because the answer is 303	Using the number 4,236, how many numbers up to 20 does it divide by without a remainder?	1, 2, 3, 4, 6, 12 They are all factors
Violet Violet I know there will be a remainder before I start.	Violet could have partitioned the number into 3,600 and 36 to see that it is divisible by 12	Is there pattern?	of 12
Is she correct?			
Explain how you know.			

#### Factors

#### Notes and Guidance

Children understand the relationship between multiplication and division and can use arrays to show the relationship between them. They know that division means sharing and finding equal groups of amounts. Children learn that a factor of a number is the number you get when you divide a whole number by another whole number and that factors come in pairs.

(factor  $\times$  factor = product).

# Mathematical Talk

- How can work in a systematic way to prove you have found all the factors?
- Do factors always come in pairs?
- How can we use our multiplication and division facts to find factors?

# Varied Fluency

If you have twenty counters, how many different ways of arranging them can you find? How many factors of twenty have you found? E.g. A pair of factors of 20 are 4 and 5.

2 (

Circle the factors of 60

9, 6, 8, 4, 12, 5, 60, 15, 45,

Which factors of 60 are not shown?

3 Fill in the missing factors of 24  $1 \times \begin{bmatrix} -2 \\ -2 \end{bmatrix} \times 12$ 

> $3 \times \begin{bmatrix} - \\ - \end{bmatrix} \quad \begin{bmatrix} - \\ - \end{bmatrix} \times \begin{bmatrix} - \\ - \end{bmatrix}$ What do you notice about the order of the factors? Use this method to find the factors of 42

### **Factors**

He pai 1 2 3	re is Ka <u>r</u> rs: 36 18 12	yla's method for finding factor Use Kayla's method to find the factors of 64	<ul><li>12 is called an abundant number because 12 is less than the sum of its factors.</li><li>How many abundant numbers can you find between 1-40? Start with the number 1 and work systematically to 40.</li></ul>	18, 20, 24, 30, 36, 40.
4 5 6	9 X 6	When do you put a cross next to a number? What do you do if a number appears twice?	Sometimes, Always, Never: An even number has an even amount of factors Sometimes, Always, Never: An odd number has an odd amount of factors	Sometimes e.g. 6 has four factors but 36 has 9 Sometimes. E.g. 21 has 4 factors
To find the factors of a number, you have to find all the pairs of numbers that		factors of a number, you have he pairs of numbers that gether to give that number		but 25 has an odd number (3),
Factors of $12 = 1, 2, 3, 4, 6, 12$ If we leave the number we started with (12) and add all the other factors together we get 16.		12 = 1, 2, 3, 4, 6, 12 the number we started with Id all the other factors e get 16.	True or False? The bigger the number, the more factors it has	This is false e.g. 12 has 6 factors but 97 only has 2.

#### Year 5 | Autumn Term | Teaching Guidance

Week 6 to 8 – Number: Multiplication and Division

#### **Common Factors**

#### Notes and Guidance

Using their knowledge of factors, children find the common factors of two numbers.

They use arrays to compare the factors of a number and use a Venn diagram to show their results.

### Mathematical Talk

- How can we find the common factors systematically?
- Which number is a common factor of any pair of numbers?
- How does a Venn diagram help to find common factors? Where are the common factors?

### Varied Fluency

Use arrays to find the common factors of 12 and 15 Can we arrange the counters in one row?

#### 

Yes- so they have a common factor of one. Can we arrange the counters in two equal rows?

#### 

2 is a factor of 12 but not of 15 so 2 is not a common factor. Continue to work through the factors systematically until you find all the common factors.



Fill in the Venn diagram to find the factors of 20 and 24.



Where are the common factors of 20 and 24? Can you use a Venn diagram to find the common factors of 9 and 15?

# **Common Factors**

#### Year 6 | Autumn Term | Teaching Guidance

Week 6 to 8 – Number: Multiplication & Division

#### **Common Factors**

#### Notes and Guidance

Children find the common factors of two numbers. Some children may still need to use arrays and other representations at this stage but mental methods and knowledge of multiples should be encouraged. They can show their results using Venn diagrams and tables.

Mathematical Talk

- How do you know you have found all the factors of a given number?
- Have you used a system?
- Can you explain your system to a partner?
- How does a Venn diagram help to find common factors?
- Where are the common factors?

#### Varied Fluency

1

What are the common factors of these pairs of numbers?

- 24 and 36
- 20 and 30
- 28 and 45



- Which number is the odd one out?
- 12, 30, 54, 42, 32, 48
- Can you explain why?



Two numbers have common factors of 4 and 9

What could the numbers be?

# **Common Factors**

There are 49 apples and 56 pears.	There will be 7 pieces of fruit in each basket because 7 is a common factor of 49 and 56	Tom has 2 pieces of string. One is 160cm long and the other is 200cm long. He cuts them into pieces of equal length	2, 5, 10 and 20 are common factors of 160 and 200
I think there will be baskets with 8 pieces of fruit in each		What are the possible lengths the string could be?	
I think there will be baskets with 7 pieces of fruit in each Who is correct?		Tahil has 32 football cards that he is giving away to his friends. He shares them equally. How many friends could Tahil have?	1, 2, 4, 8 or 16 friends.
Explain how you know.			

#### Year 5 | Autumn Term | Teaching Guidance

Week 6 to 8 – Number: Multiplication and Division

#### **Inverse Operations**

#### Notes and Guidance

Children use their knowledge of the times tables up to  $12 \times 12$ 

They solve problems using their understanding of the inverse and represent times-tables in a concrete and pictorial way.

Mathematical Talk

Why is it useful to know multiplication is the inverse of division?

How do our multiplication and division facts link to multiples and factors?

If I double the number I am multiplying by, what will happen to my answer?

### Varied Fluency



Can you use the same representation to complete the sentences?

> \_\_\_\_ and \_\_\_\_ are factors of \_\_\_\_ \_\_\_\_ is a multiple of \_\_\_\_ and \_\_\_\_

#### Complete the missing numbers.

 $3 \times \boxed{= 21} \qquad 3 \times \boxed{= 42}$ 42

$$2 \div$$
 = 6

$$\div$$
 14 = 6 84  $\div$  = 42  $\div$  6

Use <, > or = to complete the number sentences.

$$81 \div 9 \qquad \qquad 24 \div 3$$
$$6 \times 8 \qquad \qquad 16 \times 3$$

# **Inverse Operations**

### Reasoning and Problem Solving

I am a 2-digit number.

I can be divided by 10 with no remainder but not by 20

I am a multiple of 3 but not a multiple of 4

I have 8 factors.

What number am I?

With the first and second clue, children will work out the number will be 10, 30, 50, 70 or 90 The third clue will narrow it down to 30 or 90 The final clue will reveal the answer is 30 There are 72 wheels in a playground. There are a mixture of bikes, tricycles and toy cars in the playground. There are at least 5 bikes and at least 5 cars. There are 8 tricycles.

How many bikes could there be? How many cars could there be?





There could be 6 bikes and 9 cars, 8 bikes and 8 cars, 10 bikes and 7 cars, 12 bikes and 6 cars or 10 bikes and 5 cars.

#### **Order of Operations**

#### Notes and Guidance

Children will look at different operations within a calculation and consider how the order of operations affects the answer. The following image is useful when referring to the order of operations.



### Mathematical Talk

Does it make a difference if you change the order in a mixed operations calculation?

What would happen if we did not use the brackets?

Would the answer be correct?

### Varied Fluency

Sarah had 7 bags with 5 sweets in each. She added one more to each bag. Circle the calculation below that shows the correct working out.

7 (5 + 1) = 42 $7 \times 5 + 1 = 36$  $7 \times 5 + 1 = 42$ 

2

Daniel completed the following calculation and got the answer 168

 $2(30 \div 5) + 14 = 168$ 

Can you explain what he did and where he made the mistake?

3

Add brackets and the missing numbers to complete

$$3 + \bigcirc x 5 = 25 - 6 \times \bigcirc =$$

# **Order of Operations**

Play Countdown. Big numbers: 25, 50, 75, 100 Small numbers: 1 – 10 Without looking at the number cards, children choose 6 cards from across the big and small number cards.	Possible example: Cards chosen: 75, 25, 2, 5, 6, 10 Target number: 458 Calculation:	<ul> <li>Write different number sentences using the digits 3, 4, 5 and 8 before the equals sign that use:</li> <li>One operation</li> <li>Two operations, no brackets</li> <li>Two operations with brackets</li> </ul>	Possible answers: 58 - 34 = $58 + 3 \times 4 =$ 5(8 - 3) + 4 =
Reveal a target number.	10 - 2 + (75 × 6)		
Children aim to make the target number.			