## Years 5/6

## Small Steps Guidance and Examples

## Block 3: Multiplication \& Division

## White RoseMaths

## Overview

## Small Steps

## Year 5

## Year 6

Multiples
Common Multiples
Multiply by 10, 100 and 1,000
Mental Calculations

- Divide by 10,100 and 1,000

2ultiply and divide by 10, 100 and 1,000
Reason from known facts
Multiply up to 4 -digits by 1 -digit

- Multiply 2-digits (1)

Multiply 2-digits (2)
Multiply 3-digits by 2-digits
Multiply 4-digits by 2-digits
Multiply 4-digits by 2-digits

Divide up to 4-digits - remainder

## Short division

(
Division using factors
Long division (1)
Long division (2)
Long division (3)
Long division (4)

- Factors
- Common factors


## Common factors

Inverse operations
Order of operations

## Multiples

## Notes and Guidance

Building on their times tables knowledge, children will find multiples of whole numbers. Children build multiples of a number using concrete and pictorial representations e.g. in an array.

## Mathematical Talk

What do you notice about the multiples of 2? What is the same about them, what is different?

Look at multiples of other numbers; is there a rule that links them?

## Varied Fluency

1 Circle the multiples of 5 .

| 25 | 32 | 54 | 40 | 175 | 3000 |
| :--- | :--- | :--- | :--- | :--- | :--- |

What do you notice about the multiples of 5 ?

2 Write all the multiples of 4 between 20 and 80 .

3 Roll 2 die (1-6), multiply the numbers.
What is the number a multiple of?
Is it a multiple of more than one number?

How many different numbers can you make multiples of? Can you make multiples of all numbers up to 10 ? Can you make multiples of all numbers up to 20?
Use a table to show your results. Multiply the numbers you roll to complete the table. An example is shown below

## Multiples

## Reasoning and Problem Solving

Use the digits $0-9$. Choose 2 digits.
Multiply them together.
What is your number a multiple of?
Is it a multiple of more than one number?

Can you find all the numbers you could make?

Use the table below to help.

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 |  |  |  |  |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |  |
| 7 |  |  |  |  |  |  |  |  |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |
| 9 |  |  |  |  |  |  |  |  |  |  |

Always, Sometimes, Never
The product of two even numbers is a multiple of an odd number.

The product of two odd numbers is a multiple of an even number.

Clare's age is a multiple of 7 and is 3 less than a multiple of 8 .
She is younger than 40 .
How old is Clare?

Clare is 21 years
Always- Two even
numbers multiplied together are all multiples of 1 .

Never- Two odd numbers multiplied together are always a multiple of an odd number. You cannot make a multiple of an even number.
old,

## Common Multiples

## Notes and Guidance

Building on knowledge of multiples, children find common multiples of numbers. They should continue to use a visual representation to support their thinking. They also use more abstract methods to calculate the multiples and use numbers outside of times tablefacts.

## Varied Fluency

1 On a 100 square, shade the first 5 multiples of 7 and then the first 8 multiples of 5
What do you notice?
Choose 2 other times tables which you think will have more than 3 common multiples.

2 List 5 common multiples of 4 and 3

3 Jim and Nancy play football at the same local football pitches. Jim has plays once every 4 days and Nancy plays once every 6 days. In a fortnight, how many times will they play football on the sameday?

## Common Multiples

## Reasoning and Problem Solving

| Work out the headings for the Venn  <br> diagram. Headings: <br> Multiples of 4 <br> Multiples of 6 <br> Add in one more number to each section.  <br> Can you think of a multiple of 6 and 8 <br> that is a square number? 144 is a multiple of <br> 6 and 8 |
| :--- |

$\left.\begin{array}{|l|l|}\hline \begin{array}{l}\text { Nancy is double her sister's age. } \\ \text { They are both older than } 20 \text { and } \\ \text { younger than } 50\end{array} & \begin{array}{l}\text { Nancy: } 42 \\ \text { Nancy's sister: } 21\end{array} \\ \text { Their ages are both multiples of } 7 & \\ \text { Work out their ages. }\end{array} \quad \begin{array}{l}\text { Platform } 1 \text { and } 2 \text { will } \\ \text { have a train leaving } \\ \text { at the same time } \\ \text { once every hour at } \\ \text { at 7am. } \\ \text { The last trains leaves at midnight. }\end{array} \begin{array}{l}\text { Platform } 1 \text { has a train leaving from it } \\ \text { every } 12 \text { minutes. } \\ \text { Platform } 2 \text { has one leaving from it every } \\ 5 \text { minutes. }\end{array} \begin{array}{l}\text { Therefore there will } \\ \text { be } 18 \text { times from }\end{array}\right\}$

## Multiplying by 10, 100 \& 1000

## Notes and Guidance

Children recap multiplying by 10 and 100 before moving on to multiplying by 1000. They look at numbers in a place value grid and discuss how many places to the left digits move when you multiply by different multiples of 10 .

## Mathematical Talk

Which direction do the digits move when you multiply by 10,100 or 1000 ?

How many places do you move to the left?
When we have an empty place value column to the right of our digits what number do we use as a place holder?

Can you use multiplying by 100 to help you multiply by 1000 ? Explain why.

## Varied Fluency

1 Make the number 234 on the place value grid using counters.

| HTh | Th | Th | H | T | 0 |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathrm{O}^{\circ}$ | ○○ | $\bigcirc \bigcirc$ |
|  |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |

When I multiply my number by 10 , where will I move my counters?
Remember when we multiply by $10,100,1000$, we move the digits to the left and use zero as a place holder.

2 Complete the following questions using counters and a place value grid.

$$
\begin{array}{lr}
234 \times 100= & 324 \times 100= \\
100 \times 36= & 1,000 \times 207=
\end{array}
$$

$$
45,020 \times 10=
$$

$$
=3,456 \times 1,000
$$

3 Use <,$>$ or $=$ to complete the sentences.

| $62 \times 1,000$ | $\square$ | $62 \times 100$ |
| :--- | :--- | :--- |
| $100 \times 32$ | $\square$ | $32 \times 100$ |
| $48 \times 100$ | $\square$ | $48 \times 10 \times 10 \times 10$ |

## Multiplying by 10,100 \& 1000

## Reasoning and Problem Solving

| Rosie has $£ 300$ in her bank account. | Rosie has $£ 300$ |
| :--- | :--- |
| Louis has 100 times more than Rosie in |  |
| his bank account. |  |
| How much more money does Louis have <br> than Rosie? | Louis has $£ 30,000$ <br> Louis has $£ 27,700$ |
| Emily has than Rosie. <br> and Philip has £120 in his bank account. <br> Emily says, I have ten times more <br> money than you.' Is Emily correct? <br> Explain your reasoning. | No. Emily would <br> have £1200 if this <br> was the case. |

> | Jack is thinking of a 3-digit number. | 181, 262, 343, 424, |
| :--- | :--- |
| When he multiplies his number by 100, | 505 |
| the ten thousands and hundreds digit are |  |
| the same. |  |
| The sum of the digits is 10. |  |
| What number could Jack be thinking of? |  |

## Mental Calculations

## Notes and Guidance

We have included this small step separately to ensure that teachers give emphasis to this important skill. Discussions around efficient mental calculations and sensible estimations need to run through all steps.

Sometimes children are too quick to move to computational methods, when changing the order leads to quick mental methods and solutions.

## Mathematical Talk

Is there an easy and quick way to do this?
Can you use known facts to answer the problem?
Can you use rounding?

Does the solution need an exact answer?
How does knowing the approximate answer help with the calculation?

## Varied Fluency

1 How could you change the order of these calculations to be able to perform them mentally?
$50 \times 16 \times 2=$
$30 \times 12 \times 2=$
$25 \times 17 \times 4=$
2 Jamie buys a t shirt for £9.99, socks for £1.49 and a belt for $£ 8.99$
He was charged £23.47
How could he quickly check if he was overcharged?


What do you estimate that B represents when:
$\mathrm{A}=0$ and $\mathrm{C}=1,000$
$\mathrm{A}=30$ and $\mathrm{C}=150$
$\mathrm{A}=-7$ and $\mathrm{C}=17$
$A=0$ and $C=5,000$
$A=1,000$ and $C=100,000$

## Mental Calculations

## Reasoning and Problem Solving



## Dividing by 10, 100 \& 1000

## Notes and Guidance

Children look at dividing by 10, 100 and 1000 using a place value chart. They use counters and digits to learn that the digits move to the right when dividing by powers of ten.

## Mathematical Talk

What happens to the digits?
How are dividing by 10,100 and 1,000 related to each other?
How are dividing by 10,100 and 1,000 linked to multiplying by 10, 100 and 1,000?

What does 'inverse' mean?

## Varied Fluency

1

| HTh | Th | Th | H | T | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $O$ | $O$ | $O$ |  |  |

What number is represented in the place value grid?
Divide the number by 100.
Which direction do the counters move?
How many columns do they move?
What number do we have now?
2 Complete the following using the place value grid.
Divide 460 by 10
Divide 5,300 by 100
Divide 62,000 by 1000
Divide the following numbers by 10,100 and 1000 80,000 300,000 547,000

3 Calculate $45,000 \div 10 \div 10$
How else could you write this?

## Dividing by 10, 100 \& 1000

## Reasoning and Problem Solving

David has $£ 357,000$ in his bank. He divides the amount by 1,000 and takes that much money out of the bank. Using the money he has taken out he spends £269 on furniture for his new house.

How much money does David have left from the money he took out?

Show your workings out.
Apples weigh about 160 g each.
How many apples would you expect to get in a 2 kg bag?

Explain your reasoning.

$357,00 \div 1,000=357$
If you subtract $£ 269$, he is left with $£ 88$

Children need to be able to use knowledge of equivalent measures to convert 2 kg to 2,000g.
There are approximately 12 apples.


Can you write at least two questions for each answer involving dividing by 10 , 100 or 1000 ?

Match the calculation to the answer:

| $64, \quad 640$, | 6,400 |
| :---: | :---: |
| $64,000 \div 10$ | $640 \div 10$ |
| $640,000 \div 1000$ | $6,400 \div 100$ |
| $6400 \div 10$ | $64,000 \div 1000$ |
| $64,000 \div 100$ | $640,000 \div 10$ |

How do you know? Do any of the calculations have the same answers? Is there an answer missed out? Explain what you have found.

```
Possible solutions
could be:
3970\div10=397
57,000\div10=5,700
397,000\div1000=397
40,500\div100=405
620,300\div100=6,203
The missing answer
is 64,000. Children
could use place
value grids to
demonstrate the
digits moving
columns.
```


## Multiples of 10,100 \& 1000

## Notes and Guidance

Children have been taught how to multiply and divide by 10 , 100 and 1000.

They now use knowledge of other multiples to calculate related questions.

## Mathematical Talk

If we are multiplying by 20 , can we break it down into two steps and use our knowledge of multiplying by 10 ?

How does using multiplication and division as inverses help us use known facts?

## Varied Fluency

1) $36 \times 5=180$

Use this fact to solve the following questions:

| $36 \times 50=\square$ | $500 \times 36=\square$ | $180 \div 5=\square$ |
| :--- | :--- | :--- |
| $5 \times 360=\square$ | $360 \times 500=\square$ | $1800 \div 5=\square$ |

(2) Here are two methods to solve $24 \times 20$

| Method 1 |
| :---: | :---: |
| $24 \times 10 \times 2$ |
| $=240 \times 2$ |
| $=480$ | | Method $24 \times 2 \times 10$ |
| :---: |
| $=48 \times 10$ |
| $=480$ |

What is the same about the methods, what is different?
3 Use the division diagram to help solve the calculations.
$7,200 \div 200=36$


$$
\begin{aligned}
& 3,600 \div 200= \\
& 18,000 \div 200=\square \\
& 5,400 \div \square=27 \\
& =6,600 \div 200
\end{aligned}
$$

## Multiples of 10,100 \& 1000

## Reasoning and Problem Solving

 working out.


Is he correct?
Explain your answer.

Tim is not correct as he has partitioned 25 incorrectly.

He could have divided by 5 twice.

The correct answer should be 24


## Reason from Known Facts

## Notes and Guidance

Pupils should be able to use their understanding of known facts from one calculation to work out the answer of another similar calculation without starting afresh.

They should use reasoning and apply their knowledge of commutativity and inverse.

## Mathematical Talk

What is the inverse?
When do you use the inverse?
How can we use multiplication/division facts to help us answer similar questions?

## Varied Fluency

1) $70 \div-=3.5$
$70 \div=7$
$\ldots \div 2=35$
$-3.5=7$
$3.5 \times 20=$
$70 \div \ldots=3.5$
Make a similar set of calculations for $90 \div 2=45$
2) $5138 \div 14=367$

Use this to work out $15 \times 367$
3) $14 \times 8=112$

Use this to work out:
$1.4 \times 8$
$140 \times 8$

## Reason from Known Facts

## Reasoning and Problem Solving

Use this fact

## $3,565+2,250=5,815$

To work out which statements are true or false.
a) $4,565+1,250=5,815$
b) $5,815-2,250=3,565$
c) $4,815-2,565=2,250$
d) $4,065+2,750=6,315$

Write three more statements.
a) True - 1,000 added to 3,565 and 1,000 subtracted from 1,250 so cancels out
b) True - inverse
c) True - subtracted 1,000 from both numbers so difference is still 2,250
d) False - 500 have been added to both numbers so 5,815 should have increased by 1,000

## $12 \times 8$

Which of the following will give the same answer as above?
a) $3 \times 4 \times 8$
b) $12 \times 4 \times 2$
c) $2 \times 10 \times 8$

$a, b$ and $d$ will give the same answer as $12 \times 8$

## Multiply up to 4-digits by 1-digit

## Notes and Guidance

Children build on previous steps to represent a four-digit number multiplied by a one-digit number with concrete manipulatives.

Teachers should be aware of misconceptions arising from 0 in the hundreds, tens or ones column.

Children then move on to explore multiplication with exchange in first one column and then more than one column.

## Mathematical Talk

Why is it important to set out using columns?
Explain the value of each digit in your calculation.

What happens when there is a O in the ones column, tens column or hundreds column?

What do we do if there are ten counters in a column?

## Varied Fluency

1 Complete the calculation:

| Th | H | T | O |
| ---: | ---: | ---: | ---: |
| 1 | 0 | 2 | 3 |
| $\times$ |  |  | 3 |


| TH | H | T | $\bigcirc$ |
| :---: | :---: | :---: | :---: |
| - |  | (1) | (1) 1 |
| - |  | (1) 3 | (1) |
| - |  | (1) (-) | (1)(1) |

2 Write the multiplication calculation represented and find the answer.


Remember if there are ten or more counters in a column, make an exchange.
3. Sam earns $£ 1,325$ per week. How much would he earn in 4 weeks?


## Multiply up to 4-digits by 1-digit

## Reasoning and Problem Solving

Megan worked out the answer to
$1,432 \times 4$ Here is her answer:

| Th | $H$ | T | O |
| ---: | ---: | ---: | ---: |
| 1 | 4 | 3 | 2 |
| $\times$ |  |  | 4 |
| 4 | 16 | 12 | 8 |

Can you explain what Megan has done wrong?

Can you create a calculation where the answer involves:
a) An exchange in the ones column only.
b) An exchange in the tens column only.
c) An exchange in the hundreds column only.
d) More than one exchange.
e) A zero in one of the columns.

Can you work out the missing numbers using the clues?


The 4 digits being multiplied by 6 are consecutive numbers.
The first 2 digits of the answer are the same.
The 4th and 5th digits in the answer add to make the 3rd.

Answer:
$2,345 \times 5=11,725$

## Possible answers:

a) $2,317 \times 3=$

B ) $2,152 \times 4=$
c) $1,632 \times 3=$
d) $4,023 \times 5=$
e) $3,152 \times 4=$

## Multiply 2-digits (1)

## Notes and Guidance

Children use base ten to represent the area model of multiplication. The base ten will enable children to see the size and scale linked to multiplying.

Children will then move on to representing this more abstractly with place value counters alongside numbers.

## Mathematical Talk

How can we represent this calculation?
What are we multiplying?
What is happening in step 2? How have we partitioned our numbers? How does this help when multiplying?

Where can we see $20 \times 20$ ? What does the 40 represent?
What's the same and what is different between the three representations (base ten, place value, grid)?

## Varied Fluency

1. Joshua uses the base ten to solve $23 \times 22=$


Can you use the base ten to solve the following calculations?

$$
32 \times 24=\square \quad 25 \times 31=\square \quad 34 \times 23=\square
$$

2. Sammy adapts the base ten method to solve $44 \times 32=$


3 Can you use place value counters and the grid to solve the following calculations?

$$
45 \times 42=\square \quad 52 \times 24=\square \quad 34 \times 43=\square
$$

## Multiply 2-digits (1)

## Reasoning and Problem Solving

| Do you agree? Convince me! |  |  | Possible response: I disagree because if you partition each number you have $20+3$ and $50+$ 7. You will have to do $20 \times 50$ and $20 \times 7$ add $3 \times 50$ and $3 \times 7$. |
| :---: | :---: | :---: | :---: |
| Ryan hasn't finished his calculation. Can you complete the missing information and record his calculation with an answer? $\qquad$ |  |  | Possible response: Ryan needs 8 more hundreds. $40 \times 40$ |
| 40 | $\because: 8 \because$ |  | $=1,600$ and he only has 800 . |
| ${ }^{6}$ | : $\because: 8 \%$ |  | His calculation is $42 \times 46=1,932$ |

Farmer Joe has a field that measures 53 m long by 25 m wide.

Farmer Sally has a field that measures 52 m long by 24 m wide.

Max thinks they will have the same area because the numbers have only changed by one digit each.

Do you agree? Prove it!


Children may prove this with concrete or pictorial representations.

For example: I
disagree because...
 smaller. Joe has this much extra.

## Multiply 2-digits (2)

## Notes and Guidance

Children will move on from the area method and work towards more formal multiplication.

They will start by exploring the role of the zero in the calculation and why it's important. Children should understand what is happening through each step of the calculation process.

## Mathematical Talk

What is the same about the grid, expanded and compact methods?
What is the different about the grid, expanded and compact methods?
Why is the zero important?
What numbers are being multiplied in the first line and the second line?
What do we do with an exchange? What happens if the exchange is more than 1 ?
If we know what $38 \times 12$ is, how else could we work out $39 \times 12$ ?

## Varied Fluency

1 What is the same about the methods below?
What is different?


Where can you see $20 \times 10$ in the expanded and the grid calculations? Why can't we see this in the compact method? Use the methods above to calculate:

$$
34 \times 26 \quad 58 \times 15 \quad 72 \times 35
$$

(2) Complete the following to solve the calculation:


Answer the following: $27 \times 39 \quad 46 \times 55 \quad 94 \times 49$

3 Answer the following:
What is the same and
a) 38
b) $39 \times 12$
c) $37 \times 11$
$\times 12$
-
what is different about the answers?

## Multiply 2-digits (2)

## Reasoning and Problem Solving



## Multiply 3-digits by 2-digits

## Notes and Guidance

Children will extend their multiplication skills to multiplying three-digit numbers by two-digit numbers.

They will use multiplication to find area and solve multi-step problems.

## Mathematical Talk

Why is the zero important?
What numbers are being multiplied in the first line and the second line?

What do we do with an exchange? What happens if there is an exchange in the last column we multiply in?

## Varied Fluency

1 Complete the following:

| 132 |
| :---: |
| $\times 14$ |
| $528(132 \times 4)$ |
| $1320(132 \times 10)$ |



2 Calculate the following:

$$
637 \times 24=\square \quad 573 \times 28=\square 426 \times 35=\square
$$

3 A playground is 128 yards by 73 yards. Calculate the area of the playground.


## Multiply 3-digits by 2-digits

## Reasoning and Problem Solving

Tiffany has spilt paint on her maths homework, can you work out the digits that have been covered by paint?


Pencils come in boxes of 64 and a school bought 270 boxes. The school ordered 720 boxes of handwriting pens and there are 46 pens per box. How many more pens are there than pencils?


Here are examples of Casey's maths work:

$270 \times 64=17,280720$
$\times 46=33,120$
$33,120-17,280=15,840$


Can you explain the mistakes she's made? Correct each calculation.

## Answer:

A) Casey has forgotten to write the zero to show that she is multiplying by 10 .

| 987 |
| ---: |
| $\times \quad 76$ |
| $5_{5} 9_{4} 22$ |
| $+69_{4} 990$ |
| 7,5012 |

B) Casey has not exchanged into the thousand or ten thousand columns correctly


## Multiply 4-digits by 2-digits

## Notes and Guidance

Children will build upon previous learning of multiplying a 3 digit number by a 2 digit number and apply this to 4 digit numbers.

It is important that children remember the steps to take when using this multiplication method.

## Varied Fluency

1 Use the method shown to complete.

```
\begin{array} { r } { 3 2 5 0 } \\ { \times \quad 2 6 } \\ { + 1 9 5 0 0 \square \times \square } \\ { 6 5 0 0 0 \square \times \square } \end{array}
```



2 Calculate:

$$
3,282 \times 32=\square 7,132 \times 21=\square 9,708 \times 38=\square
$$

What steps do we need to go through when using this multiplication method?

Look at the numbers in each question, can they help you estimate which answer will be the largest?

## Mathematical Talk

3 Order the following from smallest to greatest.

$$
4,458 \times 56 \quad 4,523 \times 54 \quad 4,535 \times 55
$$

## Multiply 4-digits by 2-digits

## Reasoning and Problem Solving

Can you spot and correct the errors in the calculation below?

| 2534 |
| ---: |
| $\times$23 <br> 7592 <br> + <br> $\frac{5068}{12660}$ <br> 11${ }^{2}$ |

Alex has spilt paint and covered the following calculation:


What are the missing numbers?

There are 2 errors: In the first answer, the extra ten has not been added. In the second answer, the

| place | 2534 |
| :---: | :---: |
| holder | $\begin{array}{r} \\ \times \quad 23 \\ \hline 7602\end{array}$ |
|  | 7602 |
|  | 50680 |
| missing | 58282 |

The correct answer should be:


Charlotte owns an amusement park.
Which ticket type made more money?
Shown how you know this.

| Ticket type | Price | Visitors |
| :---: | :---: | :---: |
| Mon-Thurs | $£ 23$ | 3,456 |
| Fri-Sun | $£ 28$ | 2,827 |

Possible answer
Monday - Thursday
tickets:

| 3456 |
| ---: |
| $\times \quad 23$ |
| 1,0368 |
| 69120 |
| 79488 |
| 111 |

Friday to Sunday
tickets:

| 2827 |
| ---: |
| $\times \quad 28$ |
| $2261_{5} 6$ |
| 56540 |
| 79156 |
| 1 |

Monday to Thursday makes more money
because
$79,488-79,156=332$
There is a $£ 332$
difference.

## Multiply 4-digits by 2-digits

## Notes and Guidance

Children consolidate their knowledge of column multiplication.

They use these skills to solve multi step problems in a range of contexts.

## Mathematical Talk

What is important to remember as we begin multiplying by the tens number?

How would you draw the calculation?
Can the inverse operation be used?

Is there a different strategy that you could use?

## Varied Fluency

1 Calculate

| 4267 |
| ---: |
| $\times \quad 34$ |

$5734 \times 26=$
2 Lauren made cookies for a bake sale. She made 345 cookies. The recipes stated that she should have 17 chocolate chips in each cookie. How many chocolate chips will there be altogether?

3 Work out the missing number.

$$
6 \times 35=\square \times 5
$$

## Multiply 4-digits by 2-digits

## Reasoning and Problem Solving

True or false.
a) $5,463 \times 18$ is the same as $18 \times 5,463$
b) I can find the answer to $1,100 \times 28$ by using $1,100 \times 30$ and taking away two lots of 1,100
c) $70 \div 10=700 \div 100$
a) True because multiplication is commutative so the calculation can be done in any order
b) True because they both show 28 lots of 1,100
c) True because
both
numbers have been made 10 times bigger


Place the digits in the boxes to make the largest product.


## Divide up to 4-digits (1)

## Notes and Guidance

Children build on their knowledge from Year 4 of dividing three digits by one-digit. Following on from this they divide up to four-digit numbers by a one-digit number.

They use place value counters to group their number and develop their understanding of the short division method.

## Mathematical Talk

How many groups of $\qquad$ thousands are there in $\qquad$ thousand? How many groups of $\qquad$ hundreds are there in $\qquad$ tens? hundreds? How many groups of $\qquad$ tens are there in $\qquad$ How many groups of $\qquad$ ones are there in $\qquad$ ones?

Do I need to solve both calculations to compare the divisions?

## Varied Fluency

1 Here is a method to solve 4,892 divided by 4 using place value counters and short division.


$$
\begin{aligned}
& \text { Use this method to solve the following questions. } \\
& \qquad 6,610 \div 5 \quad 2,472 \div 3 \quad 9,360 \div 4
\end{aligned}
$$

2 Mr Porter has saved $£ 8,934$ pounds. He shares it between his three grandchildren. How much does each grandchild receive?
3 Use $<>$ or $=$ to compare the statements:


## Divide up to 4-digits (1)

## Reasoning and Problem Solving

| Sam is working out 2,240 divided by 7. <br> He says you can't do it because 7 is <br> larger than all of the digits in the number. | Sam is incorrect <br> because you can <br> exchange between <br> columns. When 2 <br> Do you agree with Sam? <br> Explain your answer. <br> group into 7's, you <br> can exchange 2 <br> thousands for 20 <br> hundreds. This will <br> then divide by 7. <br> 2,240 divided by 7 <br> equals 320. |
| :--- | :--- |


| Spot the mistake. <br> Explain and correct the working. |  |  |  | There is no exchanging within the calculation when there are remainders from a division. In the hundreds the hundred should have been exchanged for ten tens. There should have been two tens exchanged into the ones. The final answer should have been 3,138 |
| :---: | :---: | :---: | :---: | :---: |
| TH | H | T | 0 |  |
|  | $\text { (10) } 10)^{(10)}$ | (10) | (1) |  |
| $\begin{array}{r} 3101 \\ 3 \longdiv { 9 4 1 4 } \end{array}$ |  |  |  |  |

## Short Division

## Notes and Guidance

Children build on their understanding of dividing up to 4 digits by 1 digit by now dividing by up to 2 digits. They use the short division and focus on division as grouping.

Teachers may encourage children to list the multiples of the number to help them solve the division moreeasily.

## Mathematical Talk

What is different between dividing by 1 digit and 2 digits? If the number does not divide into the ones, what do we do?

Do we need to round our remainders up or down? Why does the context affect whether we round up or down?

## Varied Fluency

1 Solve the divisions using short division.
$5 \longdiv { 7 2 5 } \quad 3 \longdiv { 1 9 3 8 } \quad 1 2 \longdiv { 6 0 3 6 } \quad 3,612 \div 14=$

List the multiples of the number to helpyou calculate.

2 A limousine company allows 14 people per limousine. How many limousines need to be hired for 230 people?

3 Year 6 have 2,356 pencil crayons for the year. They put them in bundles with 12 in eachbundle. How many complete bundles can be made?

## Short Division

## Reasoning and Problem Solving

Find the missing digits

$$
\frac{041 \% \mathrm{r} 3}{4 \sqrt{159}}
$$



| Work out the value of $C$ <br> (The bar models are not drawn to scale) |  |  |  |  | $4,950 \div 3=1,650$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4,950 |  |  |  |  | $1,650 \div 3=550$ |
|  | A | A |  | A | $550 \div 5=110$ |
| A |  |  |  |  |  |
|  | B | B |  | B |  |
| B |  |  |  |  |  |
| C | C | C |  | C |  |

## Divide up to 4-digits (2)

## Notes and Guidance

Children continue to use place value counters to group their number and develop their understanding of the short division method.

They start to focus on remainders and build on their learning from Year 4 to understand remainders in a context.

They do not represent remainders as a fraction at this point.

## Mathematical Talk

If I can't make a group in this column, what do I do?
What happens if we can group the one equally?
In this number story, what does this remainder mean?
When would we round the remainder up or down?
What context would we just focus on the remainder?

## Varied Fluency

1 Here is a method to solve 4,894 divided by 4 using place value counters and short division.


Use this method to solve the following questions.


2 Muffins are packed in trays of 6 in a factory. In one day a factory makes 5,623 muffins.
How many trays do they need per day?
How many full trays do they have at the end of the day?
3 For the calculation, $8,035 \div 4$, can you:
Write a number story where you have to round the remainder up. Write a number story where you have to round the remainder down.
Write a number story where you have to find the remainder.

## Divide up to 4-digits (2)

## Reasoning and Problem Solving



## If there is a

remainder of 4 when dividing by 5 , the
ones digit must be 4 or 9 .
If there is a
remainder of 1 when dividing by 2 , the number must be odd so the ones digit must be 9 .
In a multiple of 9, the digits always add up to a multiple of 9, this will help me to check if my number will have a remainder of 3 when divided by 9 . Possible answers:

129, 219, 309, 399, 489, 579, 669, 759,

$$
\begin{aligned}
& 765 \div 4=191 \text { remainder } 1 \\
& 876 \div 5=175 \text { remainder } 1
\end{aligned}
$$

Does a three-digit number descending in digits divided by the next descending digit always have remainder 1 ?

Prove your answer.

This does not always have the remainder of 1.
$543 \div 2=271 r 1$
However:
$987 \div 6=164 r 3$
$654 \div 3=218$

## Division using Factors

## Notes and Guidance

Children need to use their number sense, specifically their knowledge of factors to be able to see relationships between the divisor and dividend. Beginning with multiples of 10 and moving on will allow the children to see the relationship before progressing forward.

## Mathematical Talk

What is a factor?
How does using factor pairs help us to answer division questions?
Do you notice any patterns?
Does using factor pairs always work?
Is there more than one way to solve a calculation using factor pairs?
What methods can be used to check your working out?

## Varied Fluency

1. $780 \div 20=39$ is the same as
$780 \div 10=78$ then $78 \div 2=39$
What do you notice?
Use the same method to solve $480 \div 60$
2 Use factors to help you to answer

$$
4,320 \div 15
$$

3 Eggs are put into boxes holding a dozen. A farmer wants to put 648 eggs into boxes. How many boxes will he have filled?


## Division using Factors

## Reasoning and Problem Solving

| Divide 1,248 by <br> - 48 <br> - 24 <br> - 12 <br> What did you do each time? Explain your strategy. | $\begin{aligned} & 1,248 \div 48=26 \\ & 1,248 \div 24=52 \\ & 1,248 \div 12=104 \end{aligned}$ <br> I used factor pairs to complete the first question e.g. I divided 1,248 by 12 then divided the answer by 4 Because 24 is half of 48 , I doubled 26 to get 52 <br> I repeated this with 12 to get 104 |
| :---: | :---: |
| Is Ivan correct? <br> Explain why. | Ivan is incorrect. <br> He has partitioned 15 when he should have used factor pairs e.g. 5 and 3 <br> The answer is 288 |



The children decide which factor pairs to use between:

- 2 and 12
- 4 and 6
- 10 and 14

Which will not give them the correct answer? Why?

## Long Division (1)

## Notes and Guidance

Children are introduced to long division as a different method of dividing by a 2-digit number. They divide 3digit numbers by a 2 -digit number without remainders moving from a more expanded method with multiples shown to the more formal long division method.

## Mathematical Talk

How can we use our multiples to help us divide by a 2digit number?

Why are we subtracting the totals from the beginning number (seeing division as repeated subtraction)?

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided)

## Varied Fluency

1


Solve the following divisions using the method above. Write out your multiples that may help you.

```
765\div17= 450\div15= 702\div18=
```

2 Use the long division method to solve the following calculations. One has been done for you as an example.

72
$836 \div 11=$
$798 \div 14=$
$7 \quad 2$
0

## Long Division (1)

## Reasoning and Problem Solving

| Which calculation could be the odd one | $512 \div 16=32$ <br> out below? <br> - $512 \div 16=$ <br> - $672 \div 21=$ <br> - $928 \div 29=$ <br> - $792 \div 24=$ <br> Explain why. <br> $928 \div 29=32$ <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br> Possible answers: <br> odd one out <br> because it is the <br> only 3-digit <br> number without a <br> 2 in the ones <br> column. |
| :--- | :--- |
| 792 $\div 24$ is the |  |
| odd one out |  |
| because it is does |  |
| not have the |  |
| answer 32 |  |

## Explain the mistake

$$
\begin{aligned}
& 746 \div 16= \\
& 1 6 \longdiv { 7 4 6 } \\
& \frac{-64!}{106}(\times 4) \\
& \frac{-106}{0}(\times 10)
\end{aligned}
$$

Instead of writing 10 lots of 16 as 160 they have written 10 lots of 16 as 106
This is
therefore the mistake in the calculation.

## Long Division (2)

## Notes and Guidance

Building on using long division with 3 digit numbers, children divide four digit numbers by 2 digits using the long division method.

They use their knowledge of multiples and multiplying and dividing by 10 and 100 to calculate more efficiently.

## Mathematical Talk

How can we use our multiples to help us divide by a 2 -digit number?

Why are we subtracting the totals from the beginning number (seeing division as repeated subtraction)?

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided)

## Varied Fluency

$\begin{array}{llll}0 & 4 & 8 & 9\end{array}$


2 There are 2,028 footballers in tournament. Each team has 11 players and 2 substitutes. How many teams are in the tournament?

## Long Division (2)

## Reasoning and Problem Solving

| Which question is easier and which is <br> harder? | $1,950 \div 13$ is <br> harder because <br> 13 is a prime <br> explain why. $1,950 \div 13=$ <br> number and <br> therefore cannot <br> be split into <br> factors and <br> divided in smaller <br> parts. |
| :--- | :--- |

$$
\begin{aligned}
& 6,823 \div 19=359 r 2 \\
& 8,259 \div \sqrt{2}=359 r 2 \\
& \text { Find the value of a prime number. }
\end{aligned}
$$

$$
\Sigma \sqrt{\Sigma}=23
$$

## Long Division (3)

## Notes and Guidance

Children now divide using long division where their answers have remainders. After dividing, they check that their remainder is smaller than their divisor.

Children start to understand when rounding is appropriate to use for interpreting the remainder and when the context means that this is not applicable.

## Mathematical Talk

How can we use our multiples to help us divide?
What happens if we cannot divide our ones exactly by our divisor? How do we show what we have left over?

Why are we subtracting the totals from the starting amount (seeing division as repeated subtraction)?

Does the remainder need to be rounded up or down?

## Varied Fluency

1 Elijah uses this method to calculate 372 divided by 15 . He has used his knowledge of multiples to help.


Solve the following calculations using Elijah's method. Show the multiples that you need to use to help you.

$$
271 \div 17=\quad 623 \div 21=\quad 842 \div 32=
$$

2 A school needs to buy 380 biscuits to pass around at parents' evening. They come in packets of 12 . How many packets with the school need to buy?

## Long Division (3)

## Reasoning and Problem Solving



## Long Division (4)

## Notes and Guidance

Children now divide four-digit numbers using long division where their answers have remainders. After dividing, they check that their remainder is smaller than theirdivisor.

Children start to understand when rounding is appropriate to use for interpreting the remainder and when the context means that this is not applicable.

## Mathematical Talk

How can we use our multiples to help us divide?
What happens if we cannot divide our ones exactly by our divisor?
How do we show what we have left over?

Why are we subtracting the totals from the starting amount (seeing division as repeated subtraction)?

Does the remainder need to be rounded up or down?

## Varied Fluency

1 Simon used this method to calculate 1426 divided by 13. He wrote down his multiples key facts to help him work out the answer.


2 There are 7,849 people going to a concert. Each coach holds 64 people. How many coaches are needed to transport all the people?

## Long Division (4)

## Reasoning and Problem Solving



## Factors

## Notes and Guidance

Children understand the relationship between multiplication and division and can use arrays to show the relationship between them. They know that division means sharing and finding equal groups of amounts. Children learn that a factor of a number is the number you get when you divide a whole number by another whole number and that factors come in pairs.
(factor $\times$ factor $=$ product ).

## Mathematical Talk

How can work in a systematic way to prove you have found all the factors?

Do factors always come in pairs?
How can we use our multiplication and division facts to find factors?

## Varied Fluency

1 If you have twenty counters, how many different ways of arranging them can you find? How many factors of twenty have you found? E.g. A pair of factors of 20 are 4 and 5 .

2 Circle the factors of 60

$$
9,6,8,4,12,5,60,15,45,
$$

Which factors of 60 are not shown?
3 Fill in the missing factors of 24



What do you notice about the order of the factors?
Use this method to find the factors of 42

## Factors

## Reasoning and Problem Solving



| 12 is called an abundant number <br> because 12 is less than the sum of its <br> factors. <br> How many abundant numbers can you <br> find between 1-40? Start with the <br> number 1 and work systematically to 40. | $18,20,24,30,36$, <br> Sometimes, Always, Never: An even <br> number has an even amount of factors <br> Sometimes, Always, Never: An odd <br> number has an odd amount of factors <br> Sometimes e.g. 6 <br> has four factors <br> but 36 has 9 <br> Sometimes. E.g. <br> 21 has 4 factors <br> but 25 has an odd <br> number (3), |
| :--- | :--- |
| True or False? The bigger the number, <br> the more factors it has. . | This is false e.g. 12 <br> has 6 factors but <br> 97 only has 2. |

## Common Factors

## Notes and Guidance

Using their knowledge of factors, children find the common factors of two numbers.

They use arrays to compare the factors of a number and use a Venn diagram to show their results.

## Mathematical Talk

How can we find the common factors systematically?
Which number is a common factor of any pair of numbers?
How does a Venn diagram help to find common factors? Where are the common factors?

## Varied Fluency

1 Use arrays to find the common factors of 12 and 15 Can we arrange the counters in one row?

## 00000000000000000000000000000000

Yes- so they have a common factor of one.
Can we arrange the counters in two equal rows?

## ○○○○○○ ○○○○○○○○○

2 is a factor of 12 but not of 15 so 2 is not a common factor. Continue to work through the factors systematically until you find all the common factors.
(2) Fill in the Venn diagram to find the factors of 20 and 24.


Where are the common factors of 20 and 24? Can you use a Venn diagram to find the common factors of 9 and $15 ?$

## Common Factors

## Reasoning and Problem Solving

\(\left.\left.$$
\begin{array}{|l|l|}\hline \begin{array}{l}\text { True or False? } \\
1 \text { is a factor of every number. } \\
1 \text { is a multiple of every number } \\
0 \text { is a factor if every number } \\
0 \text { is a multiple of every number }\end{array} & \begin{array}{l}\text { True } 1 \text { is a factor of } \\
\text { every number }\end{array} \\
& \begin{array}{l}\text { False } 1 \text { is only a } \\
\text { multiple of } 1\end{array} \\
\text { False } 0 \text { is only a } \\
\text { factor of } 0\end{array}
$$\right] \begin{array}{l}True 0 multiplied <br>
by any number <br>

equals 0 .\end{array}\right] .\)

I am thinking of two 2-digit numbers.
Both of the numbers have a digit total of
The numbers are 24 \& 60 .

6
Their common factors are $1,2,3,4,6, \&$ 12

What are the numbers?

## Common Factors

## Notes and Guidance

Children find the common factors of two numbers. Some children may still need to use arrays and other representations at this stage but mental methods and knowledge of multiples should be encouraged.
They can show their results using Venn diagrams and tables.

## Mathematical Talk

How do you know you have found all the factors of a given number?
Have you used a system?
Can you explain your system to a partner?
How does a Venn diagram help to find common factors?
Where are the common factors?

## Varied Fluency

1 What are the common factors of these pairs of numbers?

24 and 36
20 and 30
28 and 45

2 Which number is the odd one out?
$12,30,54,42,32,48$
Can you explain why?
3 Two numbers have common factors of 4 and 9
What could the numbers be?

## Common Factors

## Reasoning and Problem Solving



They need to be put into baskets with an equal number in each basket.


Who is correct?
Explain how you know.

There will be 7 pieces of fruit in each basket because 7 is a common factor of 49 and 56

Tom has 2 pieces of string.
One is 160 cm long and the other is 200cm long.

He cuts them into pieces of equal length.

What are the possible lengths the string could be?

Tahil has 32 football cards that he is giving away to his friends.

He shares them equally.
How many friends could Tahil have?
$2,5,10$ and 20 are common factors of 160 and 200
$1,2,4,8$ or 16 friends.

## Inverse Operations

## Notes and Guidance

Children use their knowledge of the times tables up to $12 \times 12$
They solve problems using their understanding of the inverse and represent times-tables in a concrete and pictorial way.

## Mathematical Talk

Why is it useful to know multiplication is the inverse of division?

How do our multiplication and division facts link to multiples and factors?

If I double the number I am multiplying by, what will happen to my answer?

## Varied Fluency

1 Complete the fact family.


Can you use the same representation to complete the sentences?
$\qquad$ and $\qquad$ are factors of $\qquad$
$\qquad$ is a multiple of $\qquad$ and $\qquad$
2 Complete the missing numbers.

$$
\begin{array}{ll}
3 \times \square=21 & 3 \times \square=42 \quad 42 \div \square=6 \\
\div 14=6 & 84 \div \square=42 \div 6
\end{array}
$$

3 Use $<,>$ or $=$ to complete the number sentences.
$81 \div 9$

$6 \times 8$ | $24 \div 3$ |
| :--- |
| $16 \times 3$ |

## Inverse Operations

## Reasoning and Problem Solving

| I am a 2-digit number. | With the first and |
| :---: | :---: |
| I can be divided by 10 with no remainder but not by 20 | children will work out the number will |
| I am a multiple of 3 but not a multiple of 4 | $\text { or } 90$ <br> The third clue will |
| I have 8 factors. | narrow it down to 30 or 90 |
| What number am I? | The final clue will reveal the answer is 30 |

There are 72 wheels in a playground.
There are a mixture of bikes, tricycles and toy cars in the playground.
There are at least 5 bikes and at least 5
cars.
There are 8 tricycles.

How many bikes could there be? How many cars could there be?


There could be 6 bikes and 9 cars, 8 bikes and 8 cars, 10 bikes and 7 cars, 12 bikes and 6 cars or 10 bikes and 5 cars.

## Order of Operations

## Notes and Guidance

Children will look at different operations within a calculation and consider how the order of operations affects the answer. The following image is useful when referring to the order of operations.


## Mathematical Talk

Does it make a difference if you change the order in a mixed operations calculation?

What would happen if we did not use the brackets?

Would the answer be correct?
Why?

## Varied Fluency

1 Sarah had 7 bags with 5 sweets in each. She added one more to each bag. Circle the calculation below that shows the correct working out.

$$
\begin{aligned}
& 7(5+1)=42 \\
& 7 \times 5+1=36 \\
& 7 \times 5+1=42
\end{aligned}
$$

2 Daniel completed the following calculation and got the answer 168

$$
2(30 \div 5)+14=168
$$

Can you explain what he did and where he madethe mistake?

3 Add brackets and the missing numbers to complete

$$
\begin{aligned}
& 3+\square \times 5= \\
& 25-6 \times \square=
\end{aligned}
$$

## Order of Operations

## Reasoning and Problem Solving

| Play Countdown. | Possible example: |
| :--- | :--- |
| Big numbers: 25, 50, 75,100 <br> Small numbers: $1-10$ | Cards chosen: 75, <br> $25,2,5,6,10$ |
| Without looking at the number cards, <br> children choose 6 cards from across the <br> big and small number cards. | Target number: 458 <br> Calculation: <br> $10-2+(75 \times 6)$ |
| Reveal a target number. |  |
| Children aim to make the target number. |  |

Write different number sentences using the digits $3,4,5$ and 8 before the equals sign that use:

- One operation
- Two operations, no brackets
- Two operations with brackets

Possible answers:
$58-34=$
$58+3 \times 4=$
$5(8-3)+4=$

